Junction Capacitance

Junction

charge concentration ~~N(x)~~

$N(x) \propto x^{\nu}$

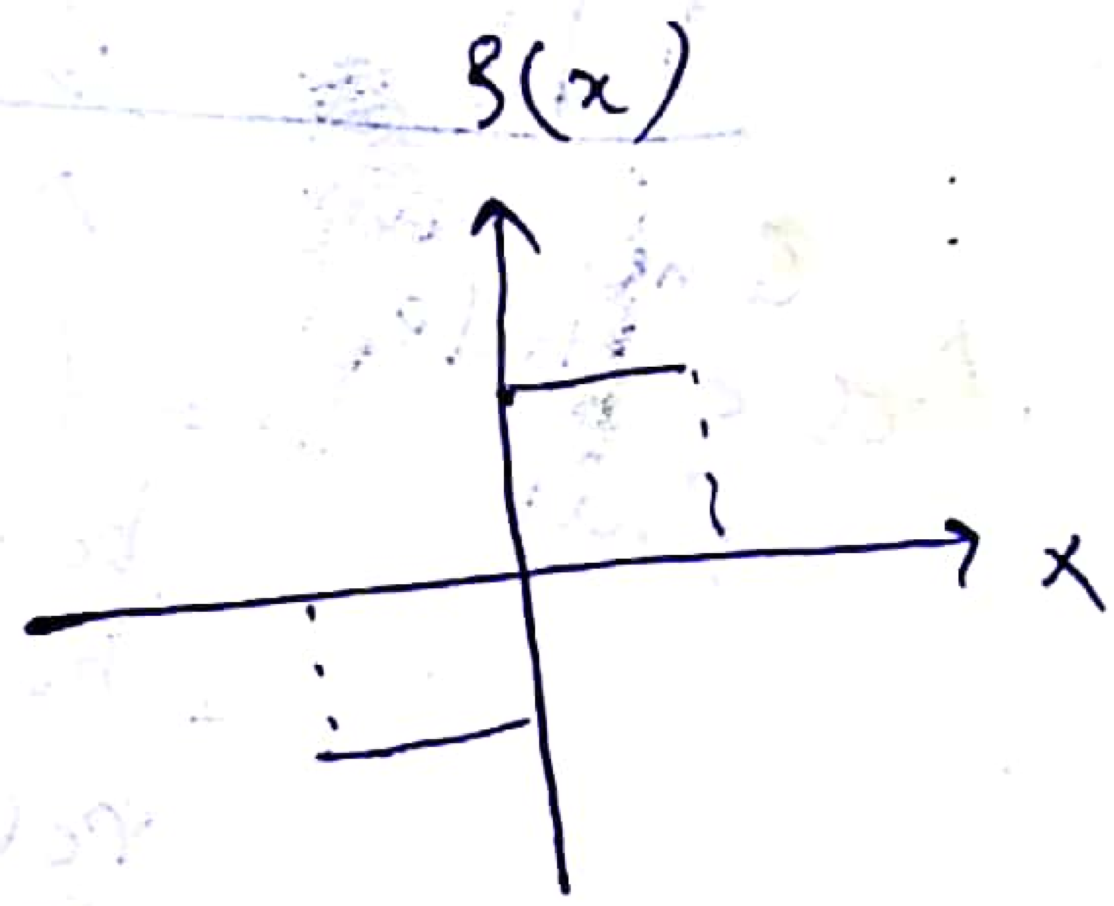
$N(x) = a x^{\nu}$

$a \to$ Graded constant.

When ① $\nu = 0 \longrightarrow$ abrupt junction

$N(x) = a$

$\rho = eN(x) = ae$

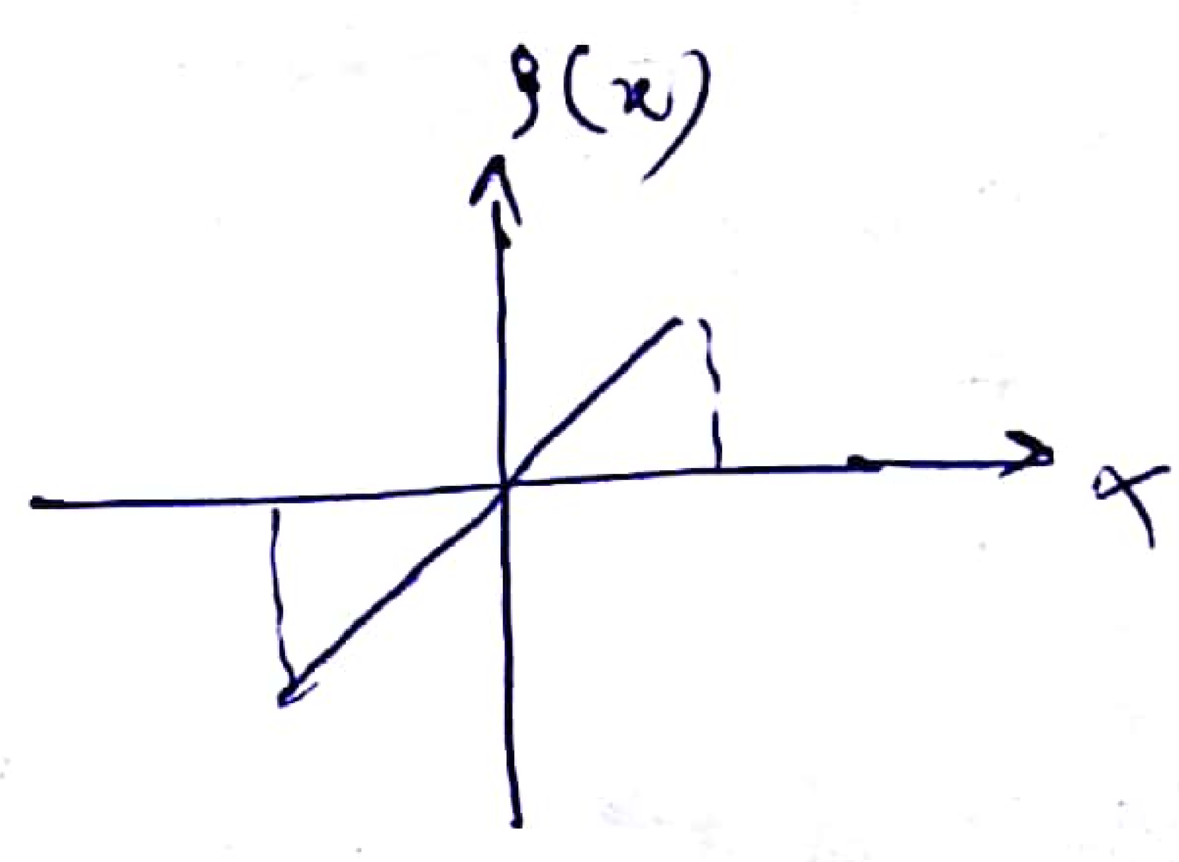


Abrupt junction

② $\nu = 1 \rightarrow$ Linearly graded junction

$N(x) = ax$

$\therefore \rho(x) = eN(x) = aex$



③ $\nu = -\frac{3}{2}$, Hyper abrupt junction

$N(x) = a x^{-3/2}$

$\rho(x) = eN(x) = aex^{-3/2}$

P-n Junction Capacitance

Junction Capacitance (C_J)

(or – Transition Capacitance,
depletion region ",
Space charge " ",
barrier ")

[* Dominant under Reverse bias condition]

Diffusion Capacitance

(or, charge Storage Capacitance)

[* Dominant under forward bias condition]

** Capacitance of a Capacitor $C = \frac{Q}{V}$ [Here charge is a ~~...~~ linear function of volt]

But Here charge ~~g~~ Q on each side of the transition region varies non linearly with applied voltage (V).

so we use more general eqn.

$$C = \left|\frac{dQ}{dV}\right|$$

Use: 1. Capacitance is the limiting factor of the usefulness of the devices.
2. ~~Tuned CKT~~ used in tuned CKT
2. useful in ~~Ca~~ CKT application
3. Provide important information about structure of the junction.

Junction Capacition [Dominant under Reverse bias]

~~The width of depletion region → increase when reverse bias voltage increase → increase~~

Reverse bias volt increase → 1. Width of depletion region increase
2. Immobile +ve charge on n side increase
3. Immobile −ve charge on p side increase

⇓

Leading Capacitive effect of the p-n junction

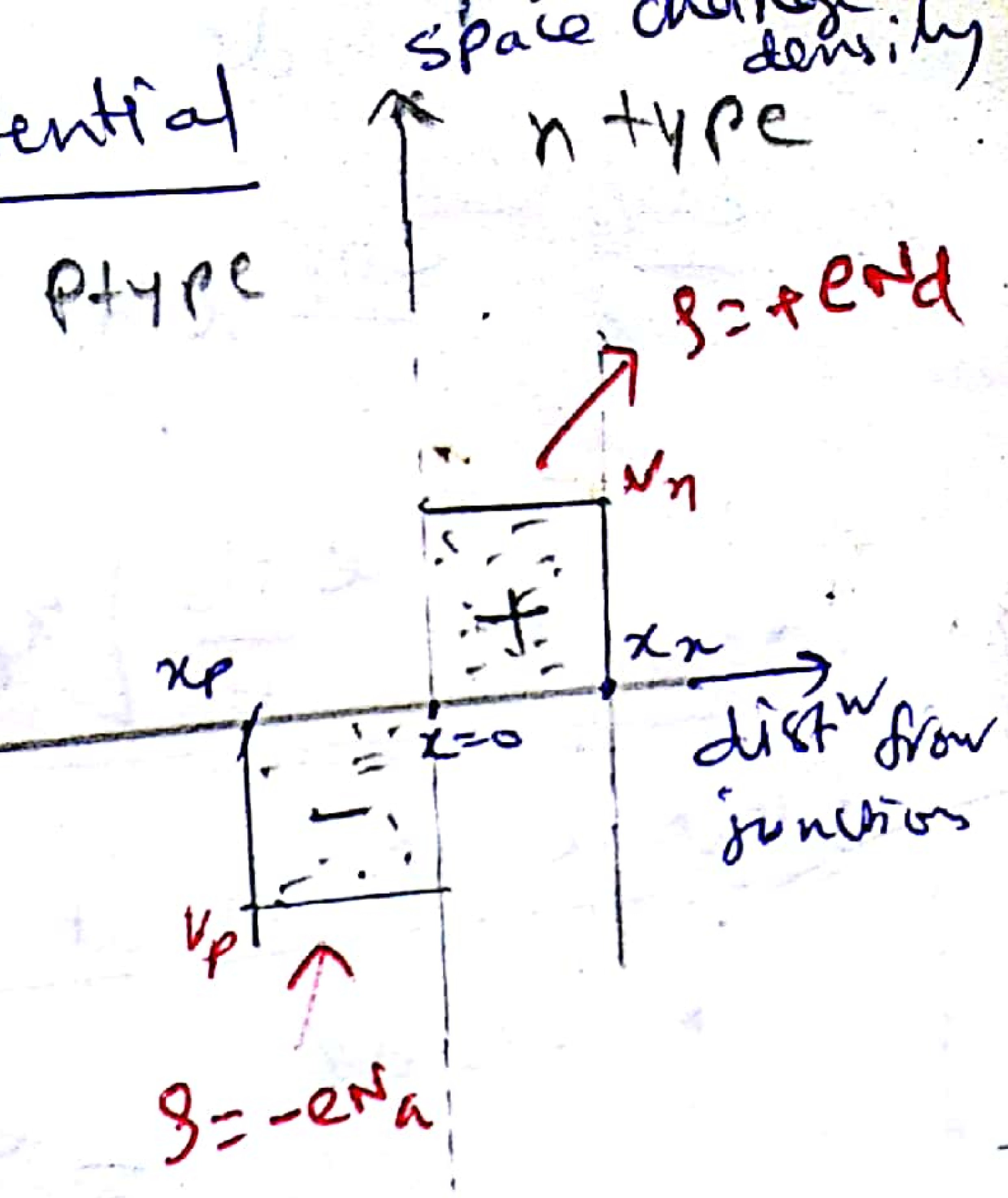
If dQ → change in charge due to voltage change dV,

Incremental capacitance $C_J = \left|\frac{dQ}{dV}\right|$

Depends on nature of p-n junction

Junction Width & Barrier Potential

Assume Abrupt junction [Symmetric junction]
(or- step graded p-n junction)



space charge density in the

P region $\rightarrow \rho = -eN_a$ (As p side is -ve)

n — " $\rightarrow \rho = +eN_d$

$V \rightarrow$ pot^n at a dist^n x from the junction, +

Poisson's equation in one dimension

Poisson

$V \rightarrow$ Potential

Poisson's eqn

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon}$$

$$\frac{d^2V}{dx^2} = \frac{eN_a}{\epsilon} \longrightarrow (1) \quad -x_p \leq x < 0$$ (for p side) charge is -ve

4 $$\frac{d^2V}{dx^2} = -\frac{eN_d}{\epsilon} \longrightarrow (2) \quad 0 < x \leq x_n$$ (for n-side charge is +ve)

Boundary conditions at $x = x_n \rightarrow \frac{dV}{dx} = 0$, & $V = V_n$

$x = -x_p \rightarrow \frac{dV}{dx} = 0$ & $V = -V_p$

$x=0$, V & $\frac{dV}{dx}$ will be continuous.

from eqn (1)
[for p side]

Integrating,

$$\frac{dV}{dx} = \frac{eN_a}{\epsilon} x + C_1$$

B.C. $\rightarrow$ at $x = x_p$, $\frac{dV}{dx} = 0$

$$\therefore -\frac{eN_a}{\epsilon} x_p + C_1 = 0$$

$$\therefore \boxed{C_1 = \frac{eN_a}{\epsilon} x_p}$$

from eqn (2)
(N side)

Integrating $$\frac{dV}{dx} = -\frac{eN_d}{\epsilon} x + C_2$$

B.C. $x = x_n$, $\frac{dV}{dx} = 0$

$$\therefore -\frac{eN_d}{\epsilon} x_n + C_2 = 0$$

$$\boxed{C_2 = \frac{eN_d x_n}{\epsilon}}$$

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(10)

P side

$$\therefore \frac{dV}{dx} = \frac{eN_a}{\epsilon} x + \frac{eN_a}{\epsilon} x_p$$

$$= \frac{eN_a}{\epsilon}\left[x_p + x\right] \quad — (3)$$

n side

$$\frac{dV}{dx} = \frac{eN_d}{\epsilon}\left[x_n - x\right] \quad — (4)$$

Electric field at p side $x<0$

$$E = -\frac{dV}{dx} = -\frac{eN_a}{\epsilon}\left[x_p + x\right] \quad —(5)$$

maximum electric field

$$E_m\,(\text{at } x=0) = -\frac{eN_a}{\epsilon} x_p$$

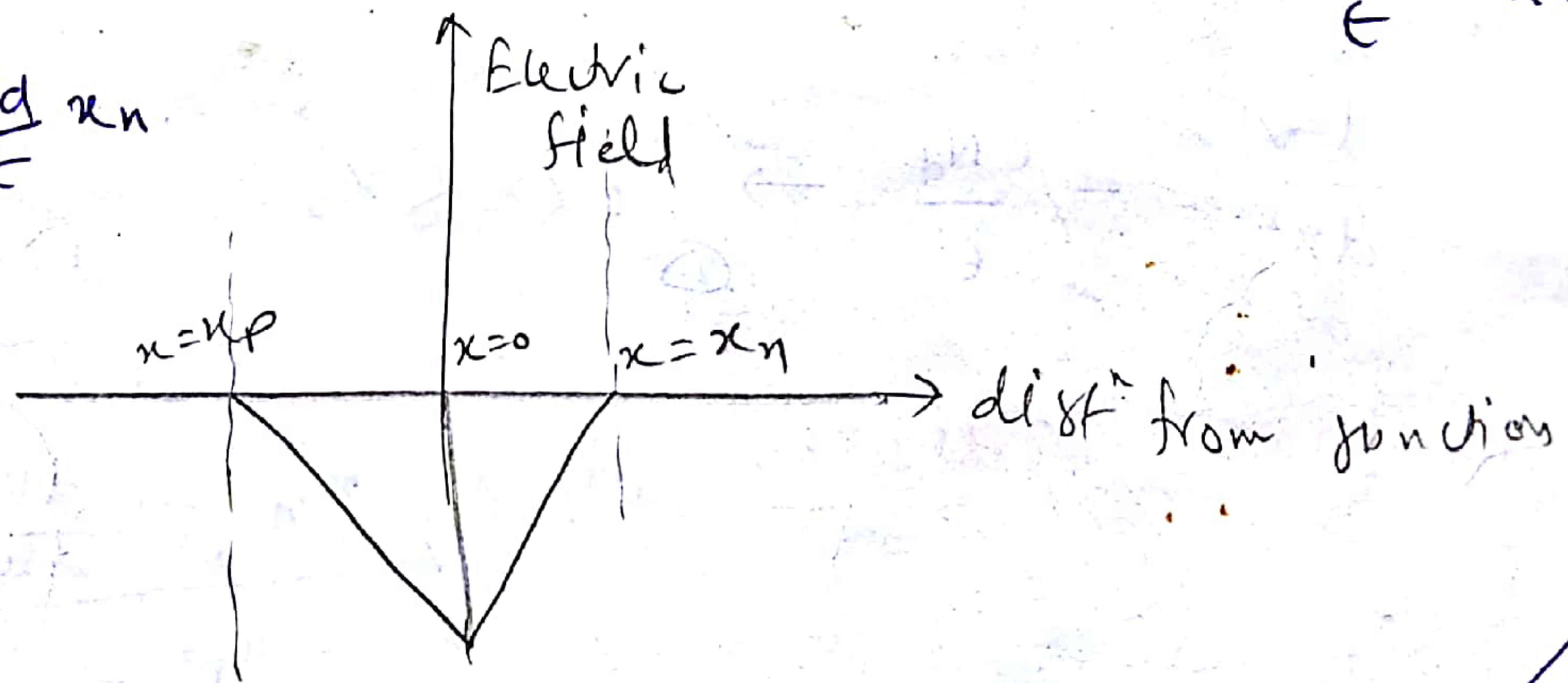
Electric field at n-side $x>0$

$$E = -\frac{dV}{dx} = -\frac{eN_d}{\epsilon}(x_n - x) \quad —(6)$$

maximum electric field

$$E_m\,(\text{at } x=0) = -\frac{eN_d}{\epsilon} x_n.$$

at $x=0$, $\frac{eN_a}{\epsilon} x_p = \frac{eN_d}{\epsilon} x_n$



P side

Integrating,

$$V = \frac{eN_a}{\epsilon_0}\left[x_p x + \frac{x^2}{2}\right] + C_3$$

B.C. at $x = -x_p$, $V = -V_p$

$$\therefore -V_p = \frac{eN_a}{\epsilon}\left[-x_p^2 + \frac{x_p^2}{2}\right] + C_3$$

$$C_3 = -V_p + \frac{eN_a}{2\epsilon} x_p^2$$

n side

Integrating

$$V = \frac{eN_d}{\epsilon}\left[x_n x - \frac{x^2}{2}\right] + C_4$$

B.C. at $x = x_n$, $V = V_n$

$$V_n = \frac{eN_d}{\epsilon}\left[x_n^2 - \frac{x_n^2}{2}\right] + C_4$$

$$\therefore C_4 = -\frac{eN_d}{2\epsilon} \cdot x_n^2 + V_n$$

$$\therefore V = \frac{eN_a}{\epsilon}\left[x_p x + \frac{x^2}{2}\right] - V_p + \frac{eN_a}{2\epsilon}x_p^2$$

$$V = -V_p + \frac{eN_a}{2\epsilon}\left[2x_p x + x^2 + x_p^2\right]$$

$$= -V_p + \frac{eN_a}{2\epsilon}\left[x_p + x\right]^2 \quad — (7)$$

$$V = \frac{eN_d}{\epsilon}\left[x_n x - \frac{x^2}{2}\right] + V_n - \frac{eN_d}{2\epsilon}x_n^2$$

$$= V_n + \frac{eN_d}{2\epsilon}\left[2x_n x - x^2 - x_n^2\right]$$

$$= V_n - \frac{eN_d}{2\epsilon}\left[x_n - x\right]^2 \quad — (8)$$

Charge neutrality condition

From B.C, at $x=0$, $\frac{dV}{dx}$ is continuous.

p side   n side

$$\frac{eN_a}{\epsilon}x_p = \frac{eN_d x_n}{\epsilon}$$

$$\Rightarrow \boxed{eN_a x_p = eN_d x_n}$$ → Condition for charge neutrality.

$$Q_p = Q_n$$

$Q_p \rightarrow$ immobile total charge on p side

$Q_n \rightarrow$ total immobile charge on n side

$$\frac{x_p}{x_n} = \frac{N_d}{N_a}$$

$$\Rightarrow \frac{x_p}{x_n + x_p} = \frac{N_d}{N_a + N_d}$$

$$\Rightarrow x_p = \frac{N_d}{N_a + N_d}W$$

$W \rightarrow$ width of the depletion layer

Similarly

$$x_n = \frac{N_a}{N_a + N_d}W$$

Again at $x=0$, Pot^n V is continuous

from eq^n (7) & (8)

$$-V_p + \frac{eN_a}{2\epsilon}(x_p^2) = V_n - \frac{eN_d}{2\epsilon}x_n^2$$

$$\Rightarrow V_n + V_p = \frac{e}{2\epsilon}\left[N_a x_p^2 + N_d x_n^2\right]$$

$\therefore$ Pot^n barrier $\phi_B = V_n + V_p$

$$= \frac{e}{2\epsilon}\left[\frac{N_a \cdot N_d^2}{(N_a + N_d)^2}W^2 + \frac{N_d N_a^2}{(N_a + N_d)^2}W^2\right]$$

$$\phi = \frac{e}{2\epsilon} \frac{N_a N_d}{(N_a + N_d)^2} [N_a + N_d] w^2$$

$$\boxed{\phi = \frac{e}{2\epsilon} \frac{N_a N_d}{(N_a + N_d)} w^2}$$

$$w^2 = \frac{2\epsilon}{e} \frac{(N_a + N_d)}{N_a N_d} \phi$$

width of the depletion layers

2) $$W = \left[\frac{2\epsilon}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \phi \right]^{1/2}$$

$$= \left[\frac{2\epsilon}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \cdot \frac{KT}{e} \ln \left(\frac{N_d N_a}{n_i^2} \right) \right]^{\frac{1}{2}}$$

We know

$$\phi = \frac{KT}{e} \ln \left(\frac{N_D N_a}{n_i^2} \right)$$

$$x_p = \frac{N_d}{(N_a + N_d)} w$$

$$= \frac{N_d}{(N_a + N_d)} \left[\frac{2\epsilon}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \cdot \frac{KT}{e} \ln \left(\frac{N_d N_a}{n_i^2} \right) \right]^{1/2}$$

$$= \left[\frac{2\epsilon KT}{e^2} \cdot \frac{N_d}{N_a (N_a + N_d)} \ln \left(\frac{N_d N_a}{n_i^2} \right) \right]^{1/2}$$

similarly

$$x_n = \frac{N_a}{(N_a + N_d)} w = \left[\frac{2\epsilon KT}{e^2} \cdot \frac{N_a}{N_d (N_a + N_d)} \ln \left(\frac{N_d N_a}{n_i^2} \right) \right]^{1/2}$$

Total charge in each portion

$$Q = eN_a x_p A \qquad A \to \text{cross-sectional Area.}$$

$$= eN_a \cdot \frac{N_d}{(N_a+N_d)} w \cdot A$$

$$= eA \frac{N_a N_d}{(N_a+N_d)} \cdot \left[\frac{2\epsilon}{e} \frac{(N_a+N_d)}{N_a N_d}\right]^{1/2} \phi^{1/2}$$

$$= \left[2\epsilon e A^2 \frac{N_a N_d}{(N_a+N_d)}\right]^{1/2} \phi^{1/2}$$

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Junction Capacitance/unit area

$$C_j = \frac{dQ}{d\phi} = \left[2\epsilon e \frac{N_a N_d}{(N_a+N_d)}\right]^{1/2} \cdot \frac{1}{2} \phi^{-\frac{1}{2}}$$

$$= \left[\frac{\epsilon e}{2} \frac{N_a N_d}{(N_a+N_d)}\right]^{1/2} \phi^{-\frac{1}{2}}$$

If the junction is forward bias then

width of the depletion layers $W_f = \left[\frac{2\epsilon}{e}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)(\phi - V)\right]^{1/2}$

Depletion layer decrease & barrier pot^n also decrease

Reverse biase:

$$W_r = \left[\frac{2\epsilon}{e}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)(\phi+V)\right]^{1/2} \to \text{increase.}$$

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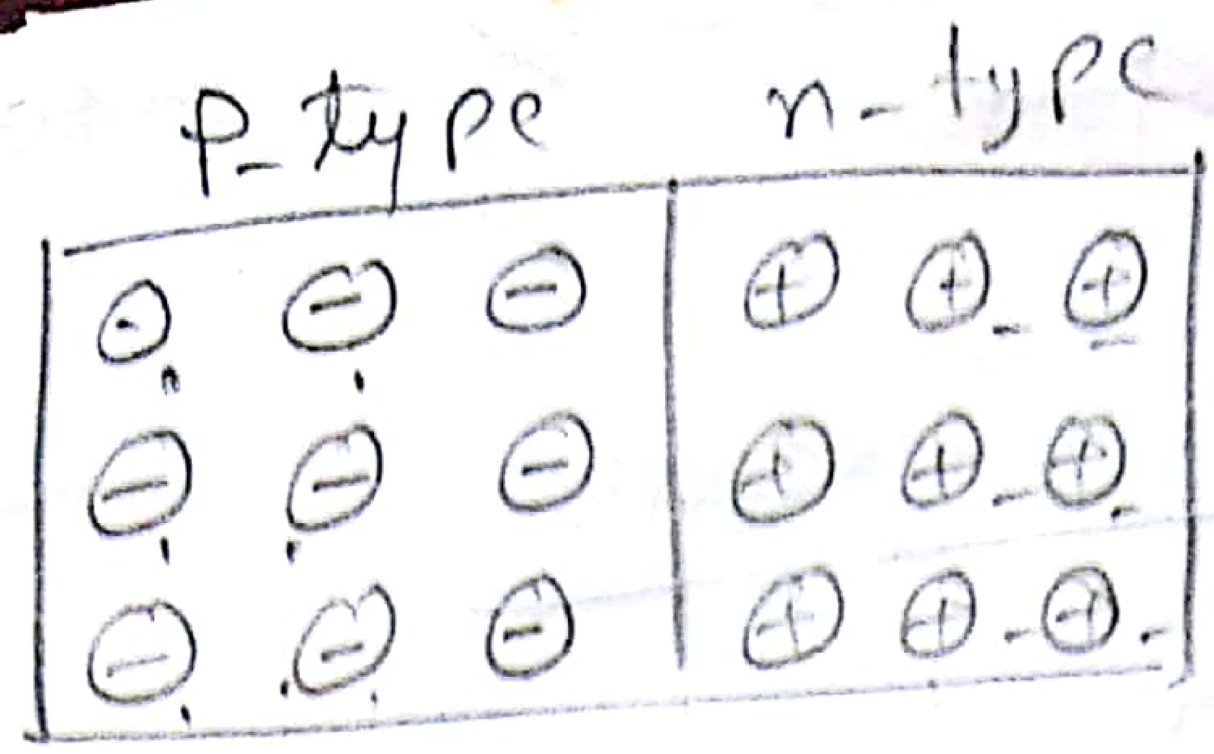
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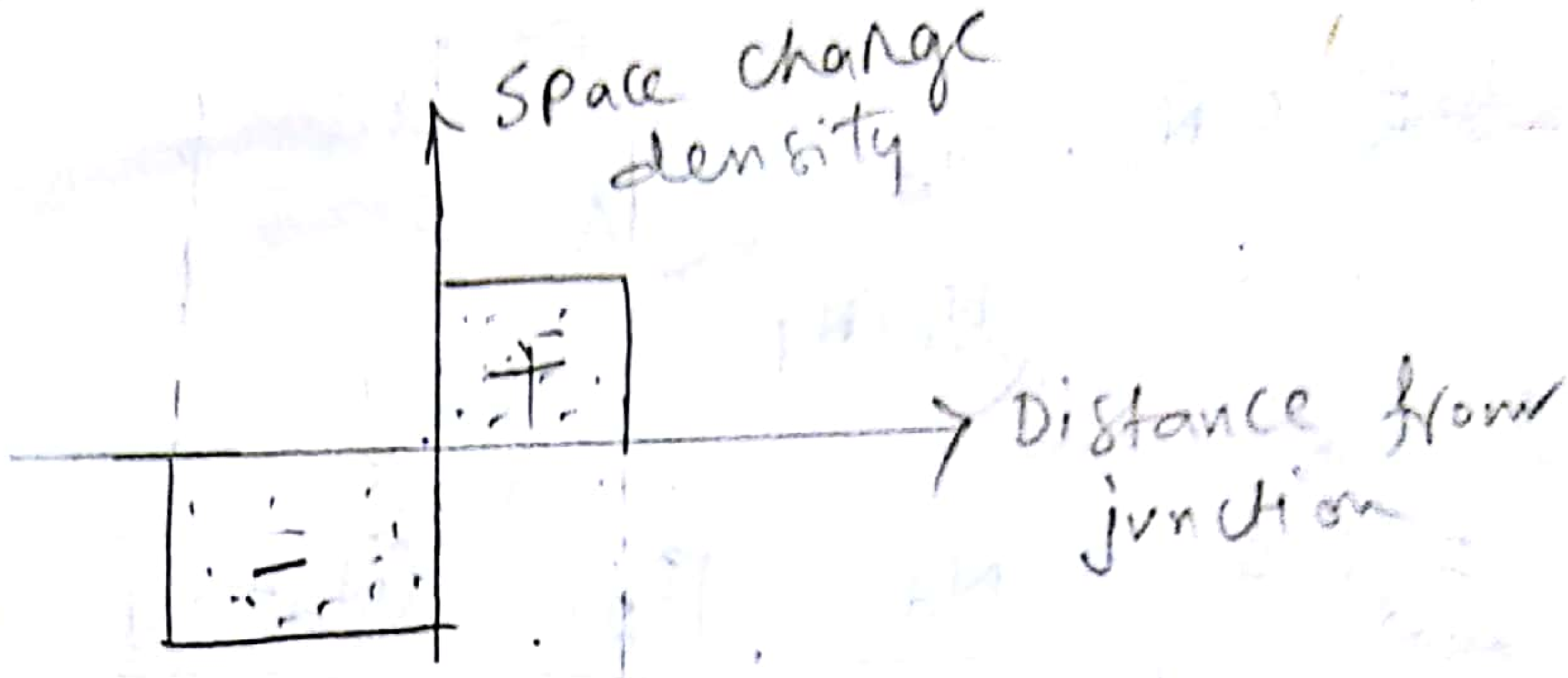
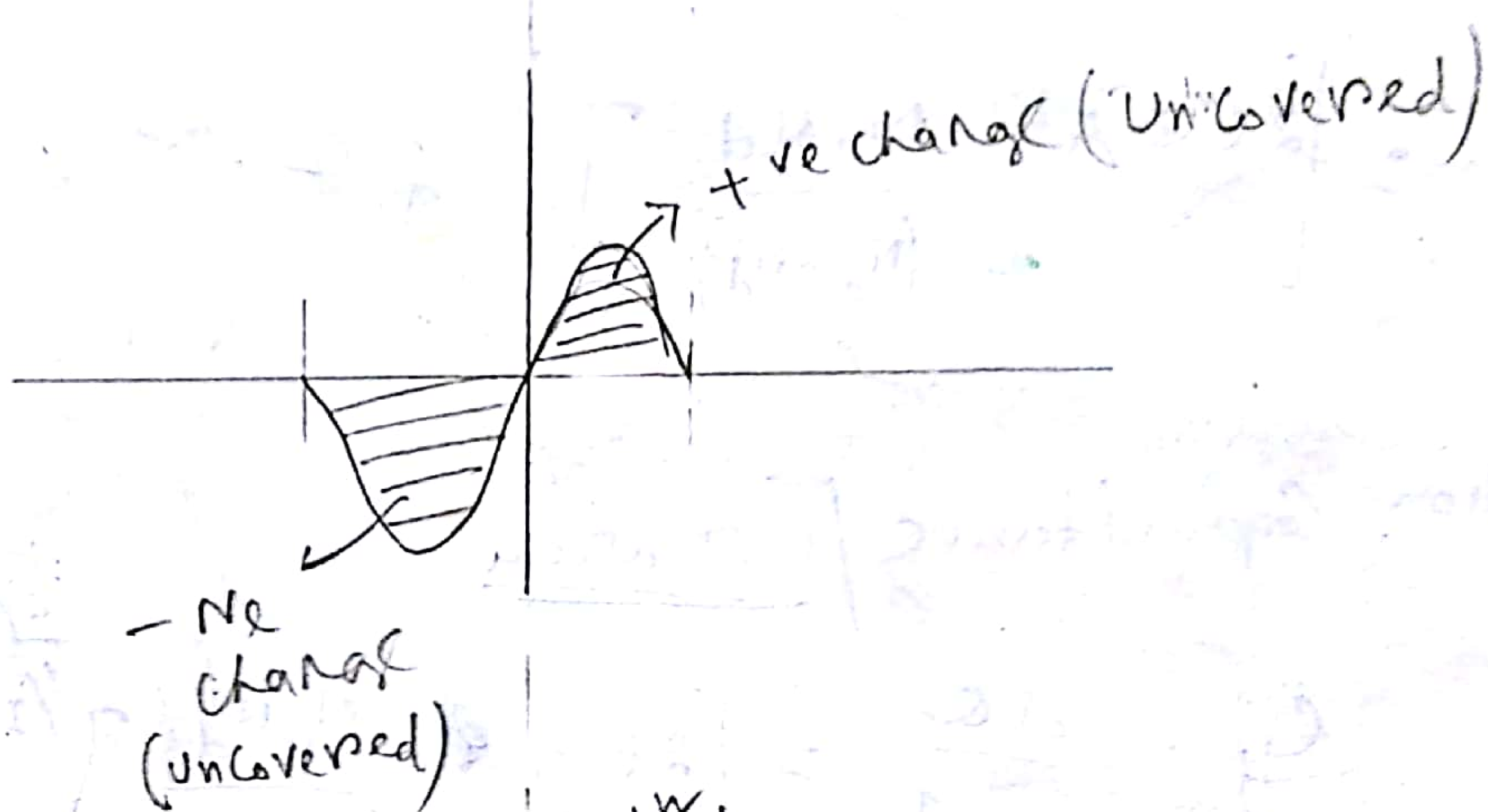
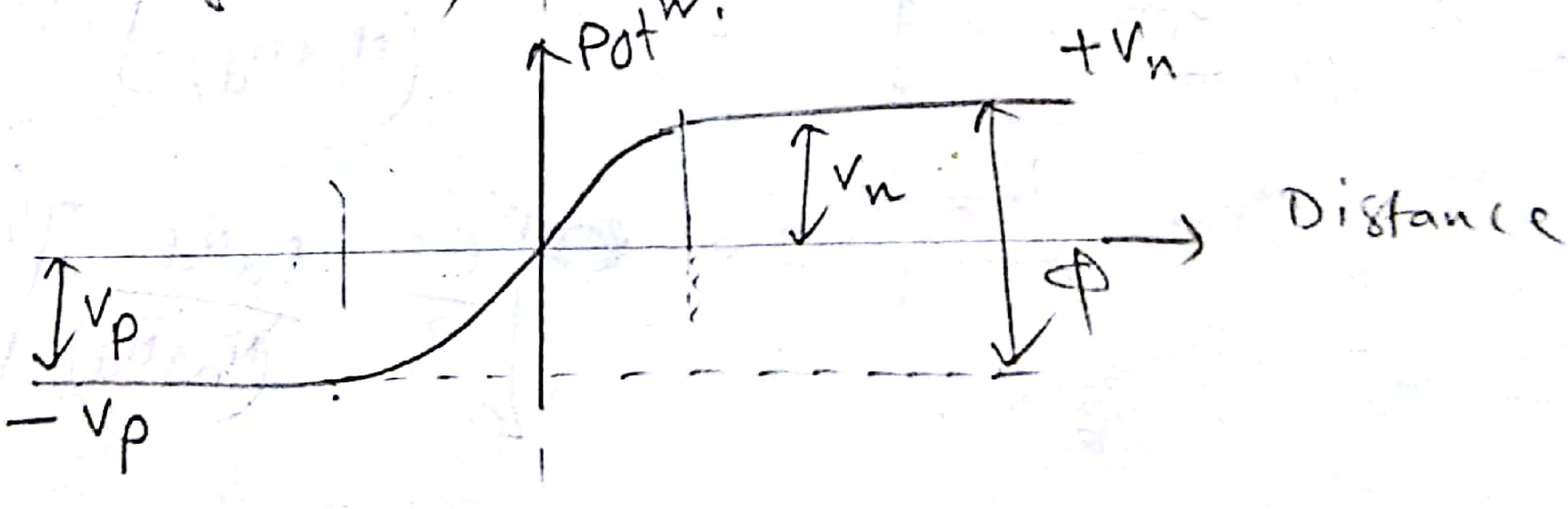
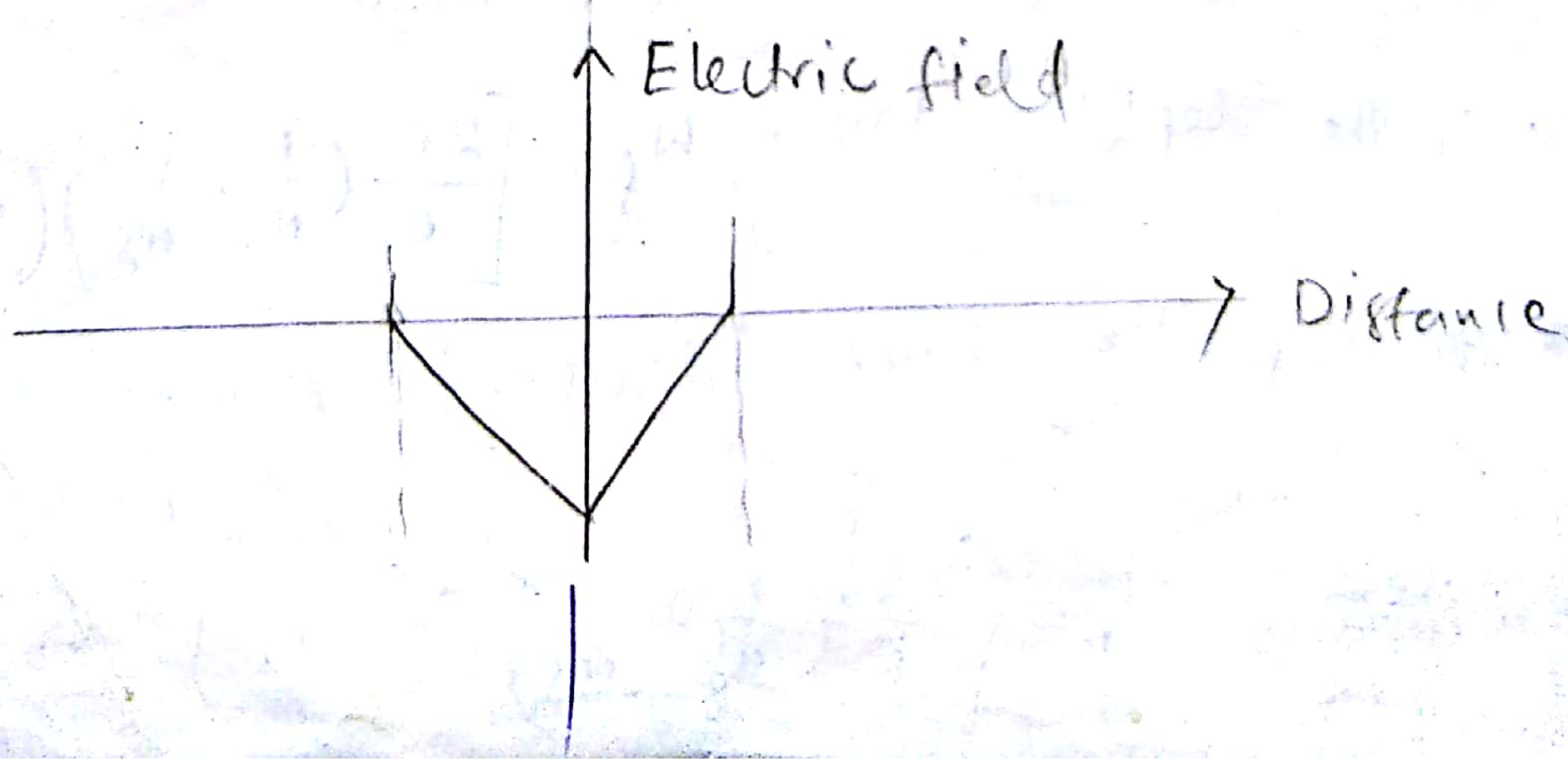
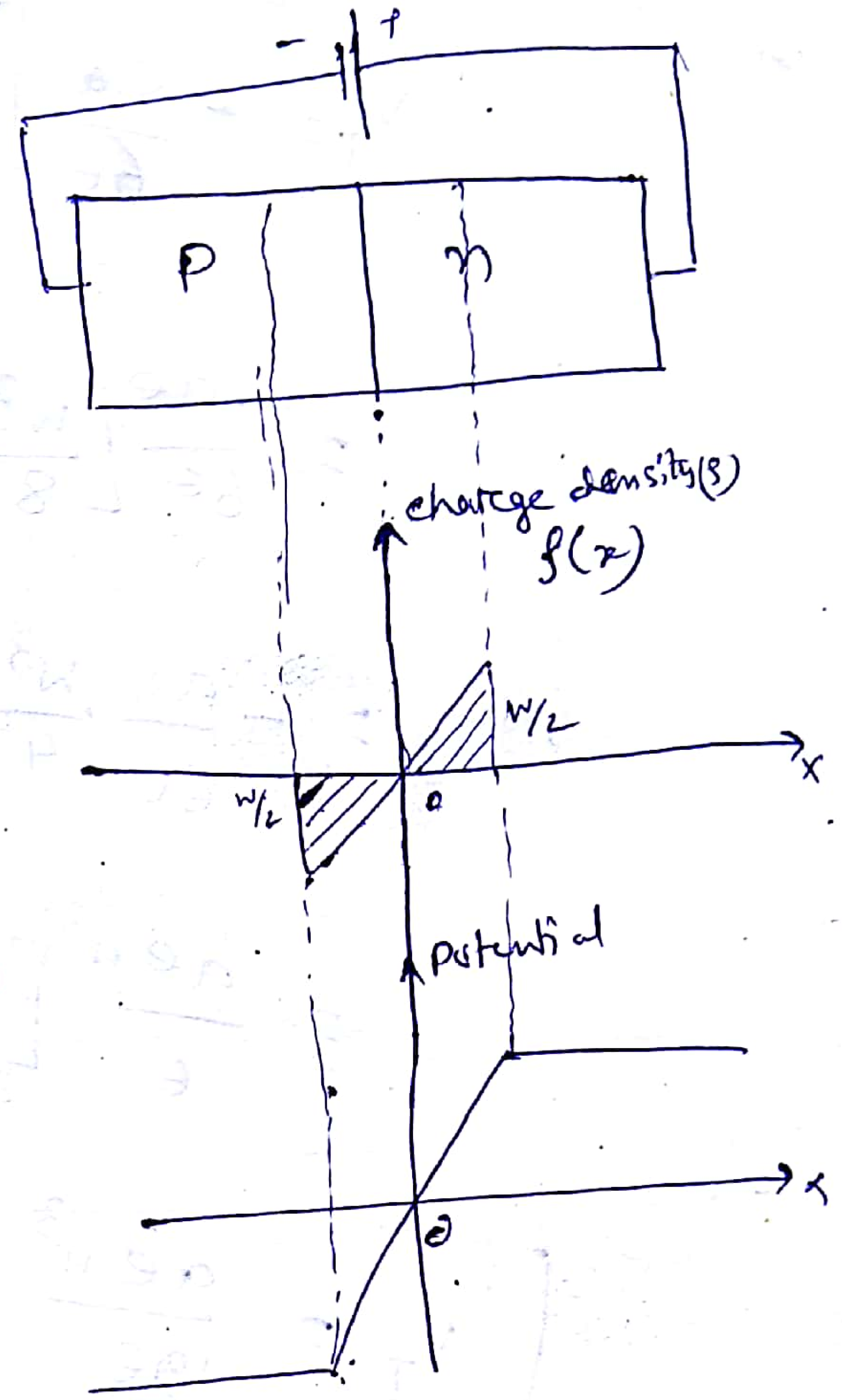

Space charge density
Distance from junction


+ve charge (uncovered)
-ve charge (uncovered)


Potⁿ.
$+V_n$
V_n
Distance
ϕ
V_p
$-V_p$


Electric field
Distance.

Linearly graded p-n-junction



Here space charge density varies → almost linearly in the transition region.

$\rho(x)$ → Space charge density within $-\frac{W}{2} < x < \frac{W}{2}$

$= e N(x)$

$$\boxed{\rho(x) = ae\,x}$$

a → graded index

$N(x) \propto x^2$

$N = ax^2$

$= ax$

$\rho = eN = aex$

Poisson's eqn in 1d. (n side)

n-side

$$\frac{d^2V}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{aex}{\epsilon}$$

Integrating, $\frac{dV}{dx} = -\frac{ae}{\epsilon}\cdot\frac{x^2}{2} + C_1$

at $x = \pm\frac{W}{2}$, electric field $\frac{dV}{dx} = 0$

$\therefore C_1 = \frac{ae}{\epsilon}\frac{W^2}{8}$

Boundary condition

at $x = \pm\frac{W}{2}$

$\frac{dV}{dx} = 0$

$$\therefore \frac{dV}{dx} = -\frac{ae}{2\epsilon}x^2 + \frac{ae}{8\epsilon}W^2$$

$$\Rightarrow \int_0^{V_T} dV = -\frac{ae}{2\epsilon}\int_{x=-\frac{W}{2}}^{W/2} x^2\,dx + \frac{ae}{8\epsilon}W^2\int_{-W/2}^{W/2} dx$$

$\int_{-V_p}^{V_N} dV$ $= V_n + V_p = V_T$

$\therefore$ ~~$V_T = \frac{ea}{8\epsilon}$~~

$$V_T = -\frac{ae}{6\epsilon}\left[x^3\right]_{-W/2}^{W/2} + \frac{aeW^2}{8\epsilon}\cdot\left[x\right]_{-W/2}^{W/2}$$

$$= -\frac{ae}{6\epsilon}\left[\frac{W^3}{8}+\frac{W^3}{8}\right] + \frac{aeW^2}{8\epsilon}\left[\frac{W}{2}+\frac{W}{2}\right]$$

$$= -\frac{ae}{6\epsilon}\cdot\frac{W^3}{4} + \frac{aeW^3}{8\epsilon}$$

$$= \frac{aeW^3}{\epsilon}\left[\frac{1}{8}-\frac{1}{24}\right]$$

$$\frac{3-1}{24} = \frac{2}{24} = \frac{1}{12}$$

$$\boxed{V_T = \frac{aeW^3}{12\epsilon}}$$

$$V_T = V_B + (V_a)$$

V_B → Pot^n barrier at equilibrium (at unbiased)

V_a → magnitude of reverse bias volt.

$$\therefore V_B + (V_a) = \frac{aeW^3}{12\epsilon}$$

Charge on one side of the depletion region is

Q = average density × volume of one side

* ~~total~~ charge on both side $= 0$.

$$= \frac{1}{2}\left[0 + ae\frac{W}{2}\right] \times \left[\frac{W}{2}\alpha\right]$$

α → Area of the junction

$$= \frac{ae\alpha}{8}W^2$$

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Junction Capacity $C_J = \frac{dQ}{dV_T} = \frac{ae\alpha}{8} 2W \frac{dW}{dV_T}$

$$= \frac{ae\alpha}{4} W \frac{dW}{dV_T}$$

Again $V_T = \frac{ae}{12\epsilon} W^3$

$$\Rightarrow W^3 = \frac{12\epsilon}{ae} V_T \longrightarrow W = \left[\frac{12\epsilon}{ae} V_T\right]^{1/3}$$

Differentiating w.r.t. V_T

$$3W^2 \frac{dW}{dV_T} = \frac{12\epsilon}{ae}$$

$$\Rightarrow \boxed{\frac{dW}{dV_T} = \frac{4\epsilon}{aeW^2}}$$

$$\therefore C_J = \frac{\cancel{ae}\alpha}{4} W . \frac{4\epsilon}{\cancel{ae}W^2} = \cancel{\alpha\epsilon\alpha} \frac{\alpha\epsilon}{W}$$

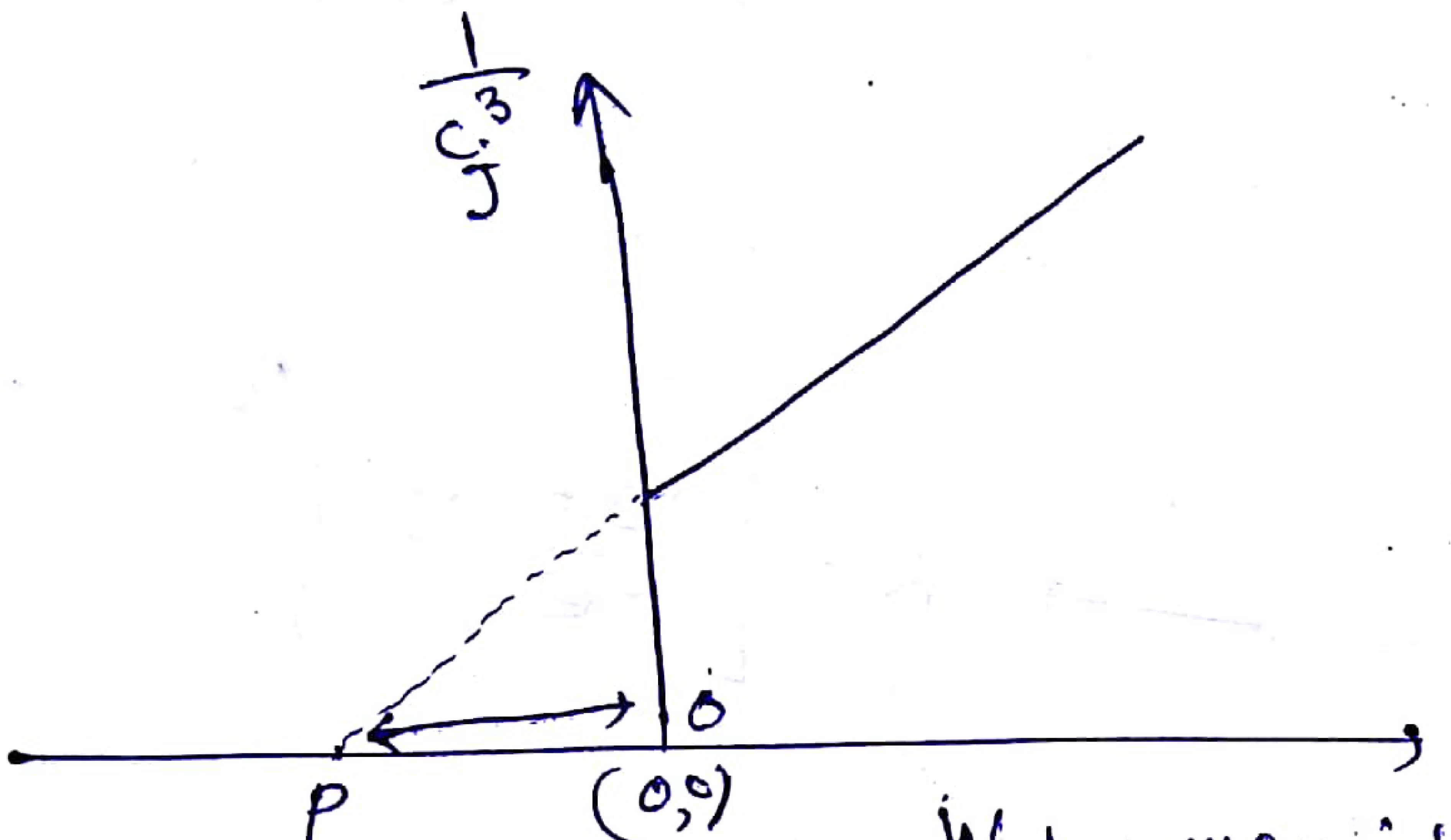
$$\boxed{C_J = \alpha\epsilon\left[\frac{ae}{12\epsilon V_T}\right]^{1/3}}$$

$\therefore$ For linearly graded p-n junction $C_J \propto \frac{1}{V_T^{3}}$

$$\therefore C_J \propto \frac{1}{[V_B + |V_a|]^3}$$

$$C_J \propto \frac{1}{[V_B + |V_a|]^3} \Rightarrow \frac{1}{C_J^3} = A[V_B + |V_a|]$$

$$= AV_B + A|V_a|$$



$|V_a| \rightarrow$ magnitude of reverse volt.

At P,

$$0 = AV_B + A|V_a|$$

$$\Rightarrow V_B = -|V_a|$$

OP → give the barrier potn at unbiassed condition.

[If we plot ~~the~~ $\frac{1}{C_J^3}$ vs $|V_a|$ graph for a linearly graded p-n junction for a fixed temp. then we may determine the ~~variation~~ value of V_B ~~for with~~ at that temperature]

Hence variation of V_B with temp. may be observed

Diffusion Capacitance (C_D)

In forward bias condition of a p-n junction $\Rightarrow$ Concentration of injected carriers changes near p-n junction

Diffusion Capacitance or Storage capacitance (C_D) = Rate of change of injected charge with the applied voltage.

At unbiased condition

$$\frac{p_p}{p_n} = e^{q\Delta\phi/KT} = e^{qV_B/KT}$$

$$\therefore \quad p_n = p_p e^{-qV_B/KT}$$

when forward bias apply

$$p_n' = p_p \, e^{-q(V_B - V_a)/KT}$$

$p_p \rightarrow$ no. of holes in p region

$p_n \rightarrow$ no. of ~~electron~~ holes " n "

[at thermal equilibrium]

$\Delta\phi = V_B \rightarrow$ Barrier pot^n at unbiased condition.

$V_a \rightarrow$ applied forward Voltage

$\therefore$ Injected holes in n-region

$$\Delta p = p_n' - p_n = p_p \, e^{-q(V_B - V_a)/KT} - p_n$$

$$= \underbrace{p_p e^{-qV_B/KT}} \cdot e^{qV_a/KT} - p_n$$

$$= p_n \, e^{qV_a/KT} - p_n$$

$$\Delta p = p_n\left[e^{qV_a/KT} - 1\right]$$ This injected holes start to recombine

The recombination is obeyed the relation

$$(\Delta p)_x = \Delta p_0 \, e^{-x/L_p}$$

$$= p_n\left[e^{qV/KT} - 1\right] e^{-\frac{x}{L_p}}$$

$L_p \to$ ~~Recomb~~ diffusion length for hole. [minority carrier]

$V \to$ applied forward voltage.

No. of holes stored/area of the n-region near junction [due to forward bias]

$$= \int_0^x \Delta p \, dx = \int_0^x p_n\left[e^{qV/KT} - 1\right] e^{-x/L_p} dx$$

$$= p_n\left[e^{qV/KT} - 1\right] \int_0^x e^{-x/L_p} dx$$

$$= p_n\left[e^{qV/KT} - 1\right] L_p$$

hole Charge stored per unit area of the n-region near junction is

$$Q_p = q \, p_n\left[e^{qV/KT} - 1\right] L_p$$

$\therefore$ Diffusion Capacitance per unit area due to hole in the n-region

$$C_{DP} = \frac{dQ_p}{dV} = \frac{q^2 p_n L_p}{KT} e^{qV/KT}$$

Similarly the diffusion capacitance per unit area due to the ~~inj~~ injection of electron in p-region.

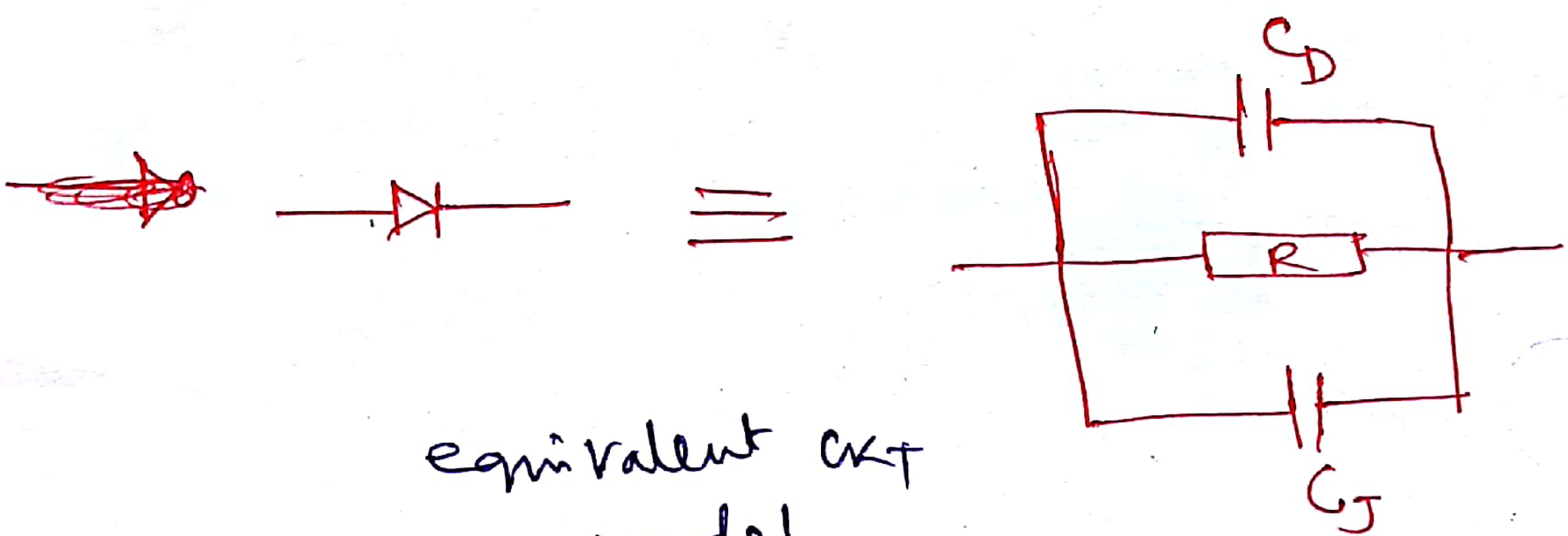
$$C_{Dn} = \frac{dQ_n}{dV} = \frac{q^2 n_p L_n}{KT} e^{\frac{qV}{KT}}$$

The two capacitance behaves as if they are ∥ to each other.

∴ Diffusion capacitance

$$C_D = C_{Dp} + C_{Dn} = \frac{q^2}{KT} e^{qV/KT} \left[p_n L_p + n_p L_n\right]$$

(#)



equivalent ckt model

The equivalent ckt model of a p-n junction diode

⇒ it includes a ~~p-n~~ non-linear p-n junction resistance (R)

+

a non-linear diffusion capacitance (C_D) and junction capacitance (C_J) both connected in ∥ with the junction resistance.