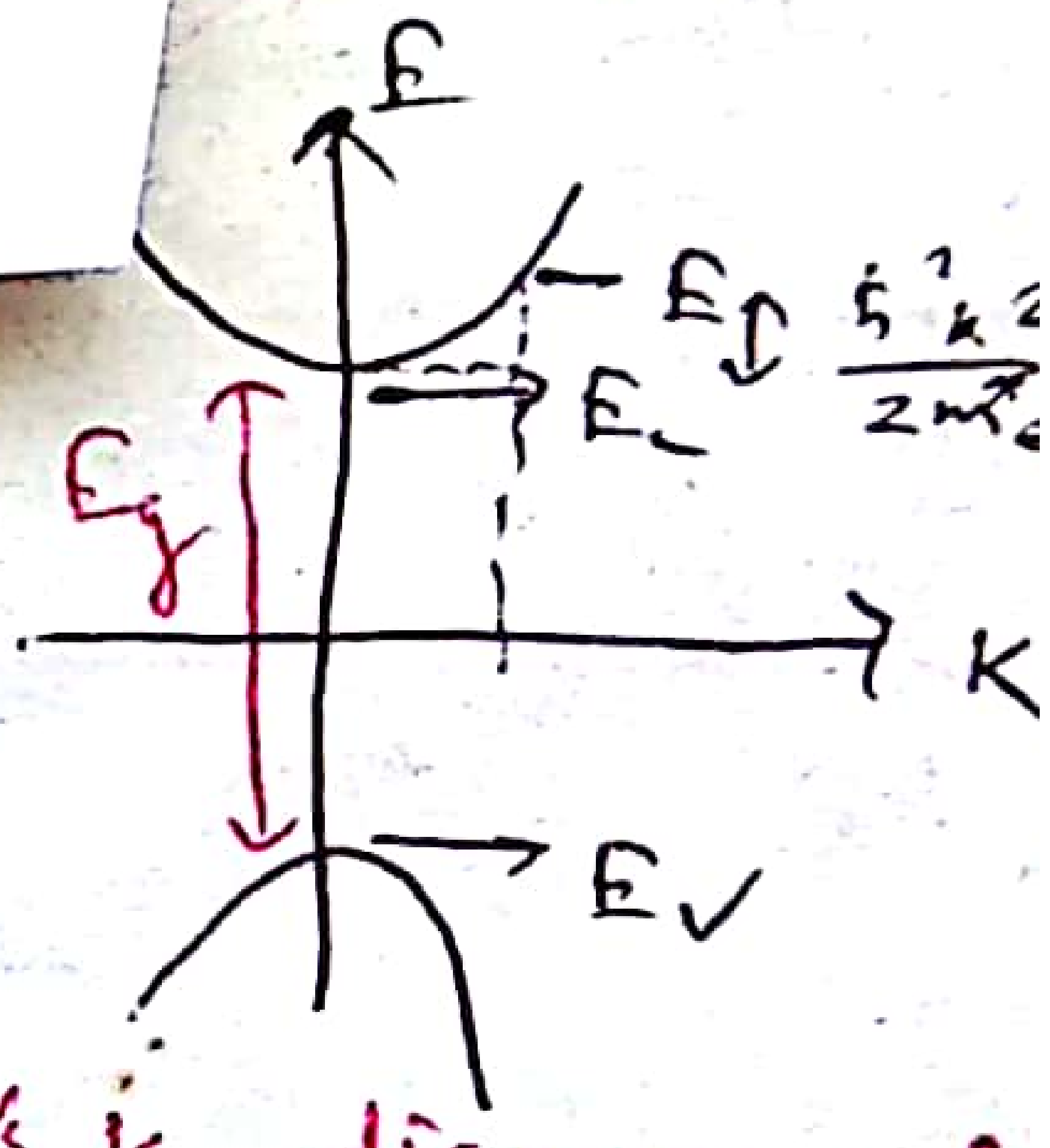
PHYSICS (PG)

COURSE NO: PHSPG-403(A)

Semi Conductor Physics and Devices



• E Vs K diagrams for electron in conduction & valence bands.

Density of states in energy between E & E+dE

Energy of an electron in the conduction band

$$E = E_C + \left(\frac{\hbar^2 k^2}{2m_e^*}\right) \longrightarrow KE$$

for simple cubic (sc) lattice of dimension L, ~~the~~ possible values of wave vector in $K = \boxed{\frac{2n\pi}{L}}$

Minimum volume of a state $= \left(\frac{2\pi}{L}\right)^3$

No. of states in the wave vector interval K & K+dK in

$$dN = \frac{4\pi k^2\, dk}{\left(\frac{2\pi}{L}\right)^3} = \frac{V k^2\, dk}{2\pi^2}$$

[where V = L^3, volume of the crys

Consider the spin of electron,

$$dN = 2 \times \frac{V k^2\, dk}{2\pi^2} = \frac{V k^2\, dk}{\pi^2}$$

No. of available state

Energy of an electron (in CB) $E = E_c + \left(\frac{\hbar^2 k^2}{2m_e^*}\right)$ → $KE = \frac{1}{2}m\dots = \frac{p^2}{2m} = \frac{1}{2m}\frac{h^2}{\lambda^2}$

$$\Rightarrow k^2 = \frac{2m_e^*}{\hbar^2}(E - E_c) \Rightarrow K = \left(\frac{2m_e^*(E-E_c)}{\hbar^2}\right)$$

$$\therefore 2k\,dK = \frac{2m_e^*}{\hbar^2}dE \longrightarrow \Rightarrow dk = \frac{m_e^*}{\hbar^2 K}dE$$

$$= \frac{m_e^*}{\hbar^2\left[\frac{2m_e^*(E-E_c)}{\hbar^2}\right]^{1/2}}dE$$

of available states in volume V.

$$\therefore dN = \frac{V}{\pi^2}\cdot\frac{2m_e^*}{\hbar^2}(E-E_c)\cdot\frac{m_e^*}{\hbar\left[2m_e^*(E-E_c)\right]^{1/2}}dE$$

$$= \frac{Vm_e^*}{\pi^2\hbar^3}\left[2m_e^*(E-E_c)\right]^{1/2}dE$$

No. of available states/volume [no. of occupied states between energy E & E+dE] in energy betn E & E+dE is

$$g(E)\,dE = \frac{m_e^*}{\pi^2\hbar^3}\left[2m_e^*(E-E_c)\right]^{1/2}dE$$

$$= \left\{\frac{4\pi(2m_e^*)^{3/2}}{h^3}(E-E_c)^{1/2}\right\}dE \Rightarrow \text{Density of States.}$$

g(E) → Energy density of states at the bottom of CB

f(E) dE the Probability of occupation

N.Degenerate & degenerate semiconductor on the basis of electron in CB.

Carrier Concentration in Conduction band (No. of electron/volume)

$$n_c = \int_{E_c}^{\infty} \cancel{g(E)f(E)\,dt}\; g(E)\,f(E)\,dE$$

f(E) ⇒ Probability of occupation of such state. (Distribution function)

[The upper limit is ∞ for convenience in the integration. the limit include all the electron in CB. ~~When~~ when E > E_f, then f(E) = 0.]

∴ Electron in solid obey Fermi-Dirac Statistics)

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

[f(E) → Probability of a state corresponding to energy E being occupied by an electron at temp. T K]

$$\therefore \; n_c = \frac{4\pi\,(2m_e^*)^{3/2}}{h^3}\int_{E_c}^{\infty}(E-E_c)^{1/2}\cdot\frac{1}{\left[1+e^{(E-E_F)/kT}\right]}\,dE$$

Put $\frac{E-E_c}{kT} = x$

$\frac{E_F-E_c}{kT} = \xi$

$dE = kT\,dx$

E	E_c	∞
x	0	∞

$$n_c = \frac{4\pi\,(2m_e^*)^{3/2}}{h^3}\int_{x=0}^{\infty}(kT)^{1/2}x^{\frac{1}{2}}\cdot\frac{1}{1+e^{x-\xi}}\,(kT)\,dx$$

$$= \frac{4\pi(2m_e^*)^{3/2}}{h^3}\cdot(kT)^{3/2}\left[\int_{x=0}^{\infty}x^{\frac{1}{2}}\,\frac{1}{e^{x-\xi}+1}\,dx\right]$$

$$= \frac{2}{\sqrt{\pi}}\underbrace{\left[2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right]}\;\varphi_{\frac{1}{2}}$$

where $\varphi_{\frac{1}{2}} = \int_{x=0}^{\infty}x^{\frac{1}{2}}\,\frac{1}{e^{x-\xi}+1}\,dx$

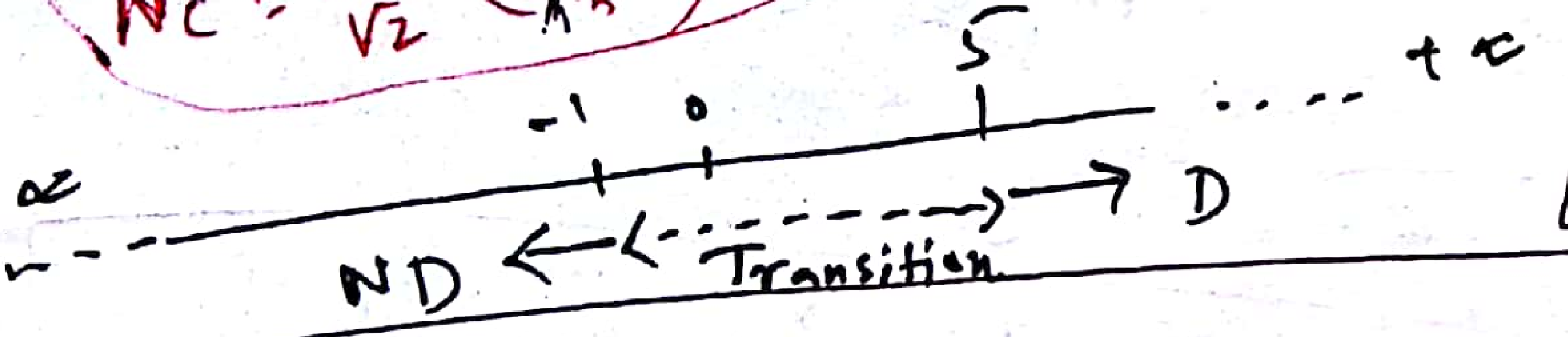
$\Rightarrow$ Fermi integral of order $\frac{1}{2}$.

$$n_c = \frac{2}{\sqrt{\pi}}\,N_c\,\varphi_{\frac{1}{2}}$$

$$\boxed{N_c = \frac{1}{\sqrt{2}}\left(\frac{m_e^* kT}{\pi\hbar^2}\right)^{3/2}}$$

N_c = effective density of states in CB

$N_c(E)$ = total no. of available electronic states (or orbital) per unit energy range at energy (E)



$$\varphi_{\frac{1}{2}} = \frac{\sqrt{\pi}}{2}\,e^{\xi} \longrightarrow \text{for } -\infty < \xi < -1$$

$\Rightarrow$ Non-degenerate

$$= \frac{\sqrt{\pi}}{2}\,\frac{1}{(0.25+e^{-\xi})} \quad \text{for } -1 < \xi < 5$$

$\Rightarrow$ Transition Region.

$$= \frac{2}{3}\,\xi^{3/2} \quad \text{for } 5 < \xi < \infty$$

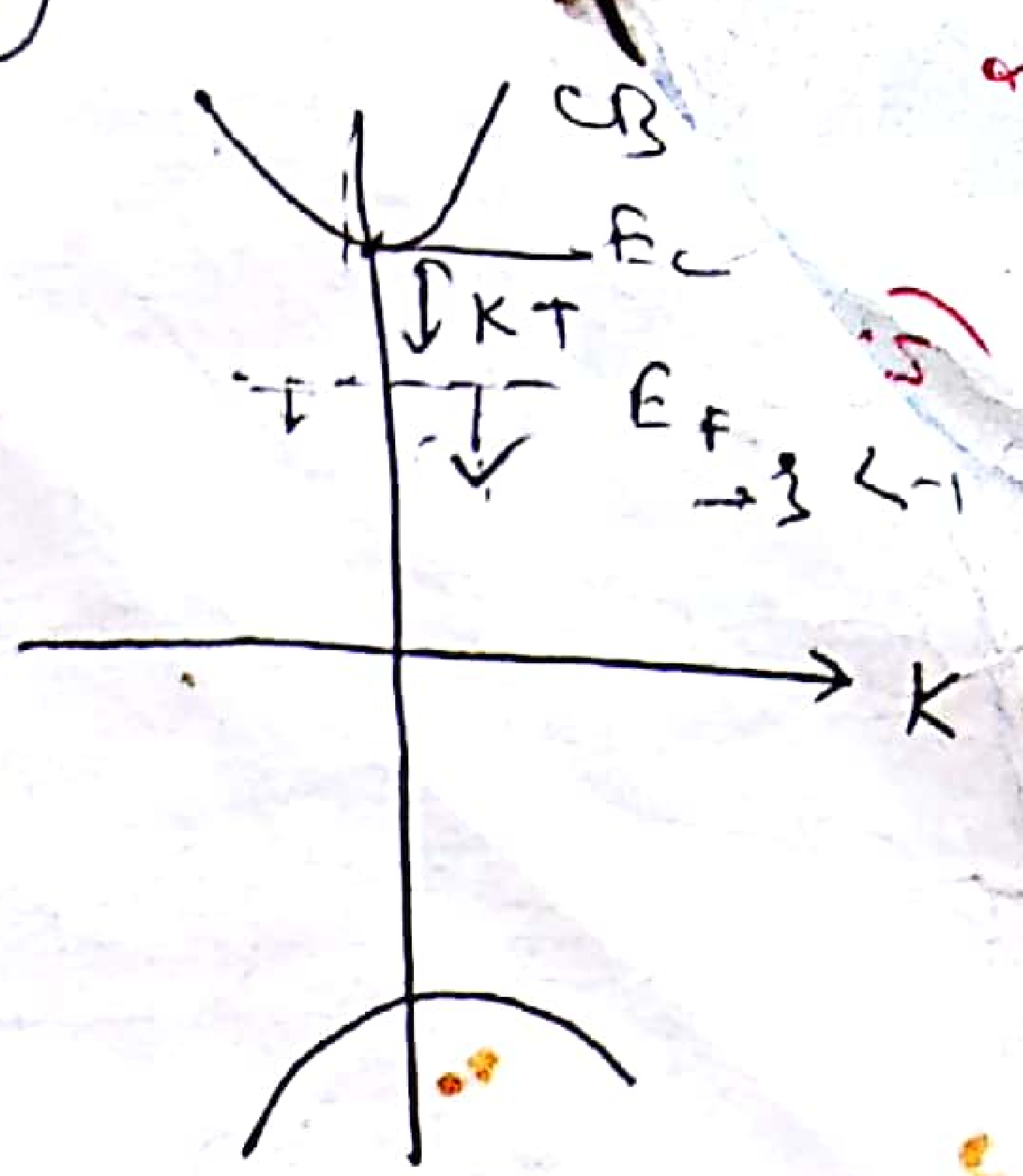
$\rightarrow$ Degenerate.

Non-Degenerate

when $\xi < -1$ $\Rightarrow$ Corresponds to Boltzman Statistics

$\Rightarrow \frac{E_F - E_c}{KT} < -1$

$\Rightarrow E_F < E_c - KT$



$$\Phi_{\frac{1}{2}} = \int_0^\infty x^{\frac{1}{2}} \frac{1}{\left(e^{x-\xi}+1\right)} dx$$

$$\frac{1}{e^{x-\xi}+1} = \frac{1}{e^{x-\xi}}$$

For all $x > 0$
ie E_c
$E > E_c$

$$= e^{\xi - x}$$

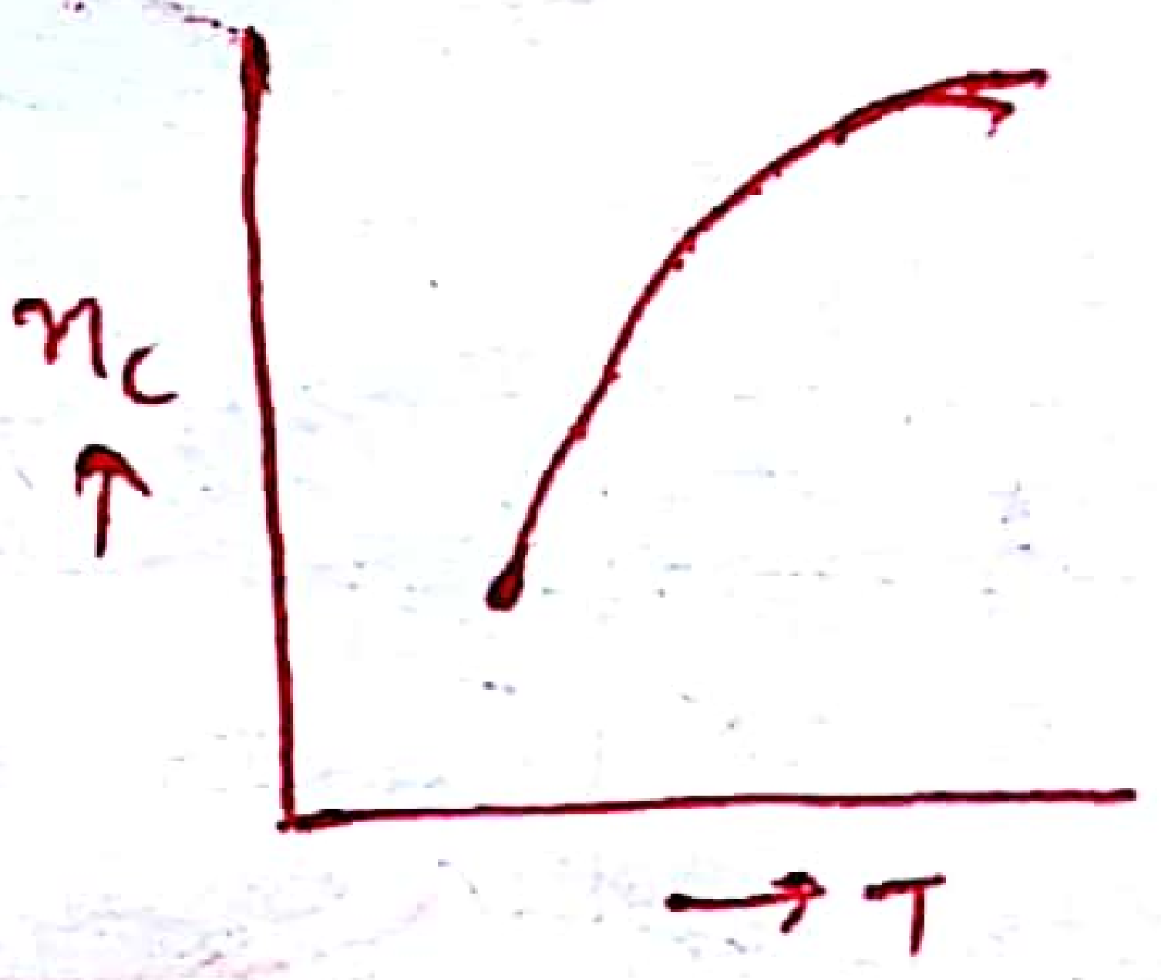
$$= \int_0^\infty x^{\frac{1}{2}} e^{\xi - x} dx$$

$$= e^{\xi} \int_0^\infty x^{\frac{1}{2}} e^{-x} dx$$

$$\boxed{\Phi_{\frac{1}{2}} = \frac{\sqrt{\pi}}{2} e^{\xi}}$$

The variation of n_c will be mainly controlled by the exponential term

At ordinary tem KT ≃ 0.025 eV.



No. of electron/volume.

$$n_c = \frac{2}{\sqrt{\pi}} N_c \cdot \Phi_{\frac{1}{2}} = \frac{2}{\sqrt{\pi}} \cdot N_c \cdot \frac{\sqrt{\pi}}{2} e^{\xi}$$

$$= N_c e^{\xi}$$

As $\xi < -1$

$n_c < N_c$

$$\boxed{n_c = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} e^{\left(\frac{E_F - E_c}{KT}\right)}}$$

Expression for Carrier Concentration in a non-degenerate semiconductor [is depend on Temp]

$E_F < E_c - KT$

i) If the Fermi level is situated below by at least KT from the conduction band than the Semiconductor will be called Non-Degenerate Semiconductor.

ii) $\xi < -1$, then $f(E) = \frac{1}{e^{(E-E_F)/kT}+1} \simeq \frac{1}{e^{(E-E_F)/kT}} << 1$

Mean that the Probability of occupation in the Conduction band for a non-degenerate semiconductor is less probable. i.e. ~~Most~~ Most of the states in CB are empty.

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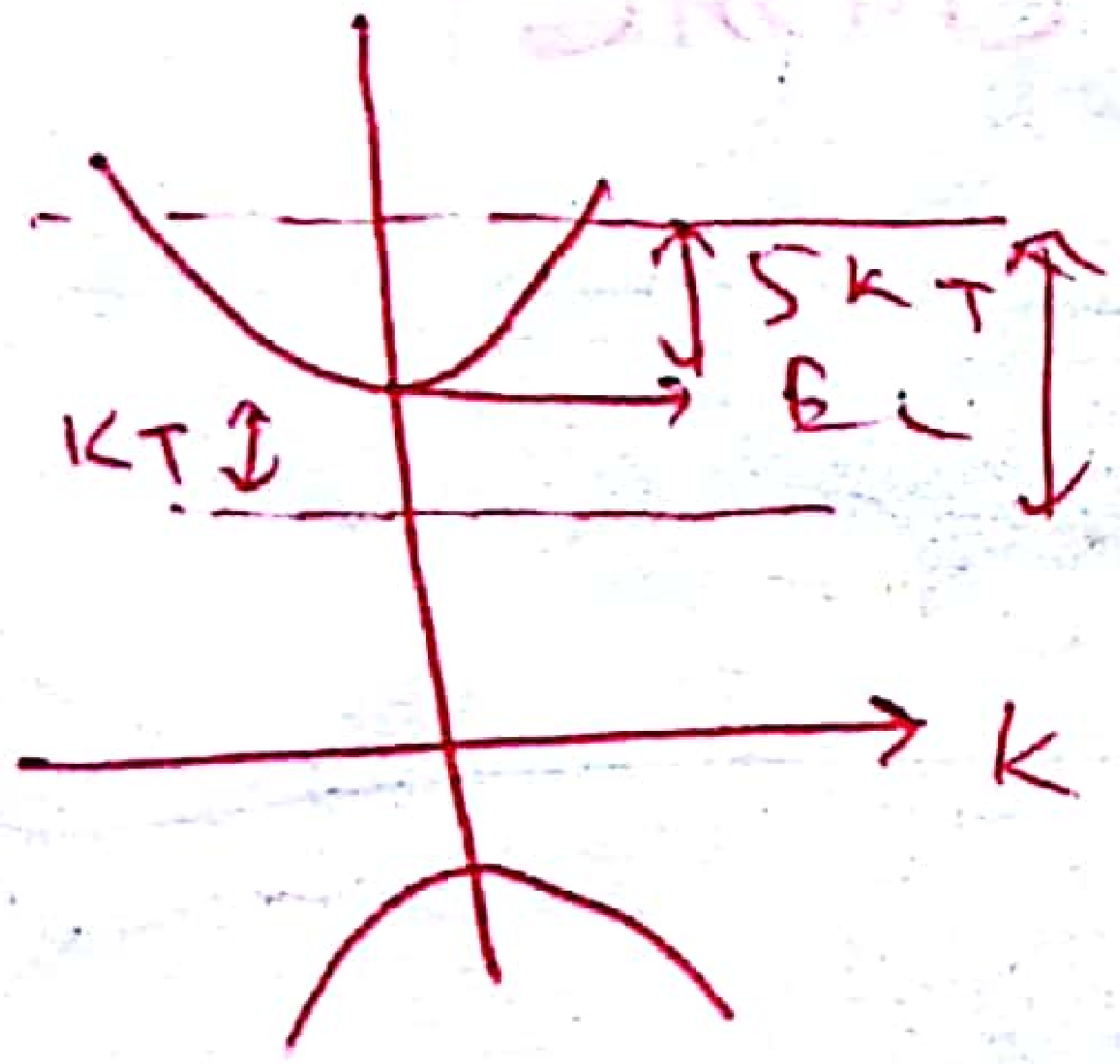
when $-1 < \xi < 5$ i.e $-1 < \frac{E_F - E_c}{KT} < 5$ $\Rightarrow E_c - kT < E_F < E_c + 5kT$



Transition region, semiconductor is either non-degenerate or degenerate semiconductor

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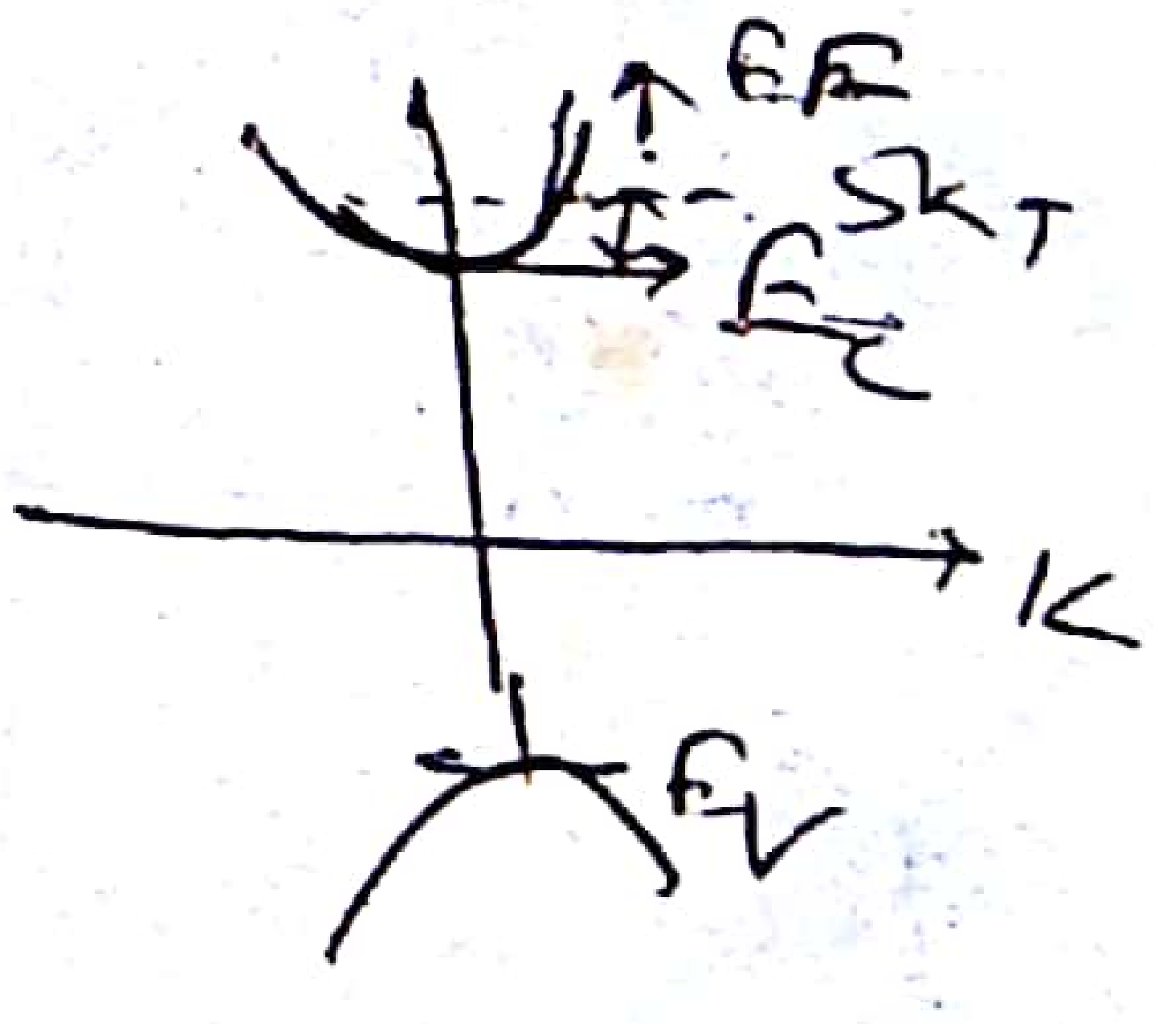
Degenerate semiconductor.

when $5 < \xi < \infty$ $\Rightarrow 5 < \frac{E_F - E_c}{KT}$ $\Rightarrow E_F > E_c + 5kT$



In this Case

$$\Phi_{\frac{1}{2}} = \int_{x=0}^{\infty} x^{\frac{1}{2}} \cdot \frac{1}{e^{x-\xi}+1}\, dx$$

[Here we cannot neglect term 1]

$$= \frac{2}{3}\xi^{\frac{3}{2}}$$

$$\boxed{n_c = \frac{2}{\sqrt{\pi}} N_c \cdot \frac{2}{3}\left(\frac{E_F - E_c}{KT}\right)^{3/2} = \frac{4}{3\sqrt{\pi}} N_c \left(\frac{E_F - E_c}{KT}\right)^{3/2}}$$

∴ carrier concentration at CB for degenerate semi.

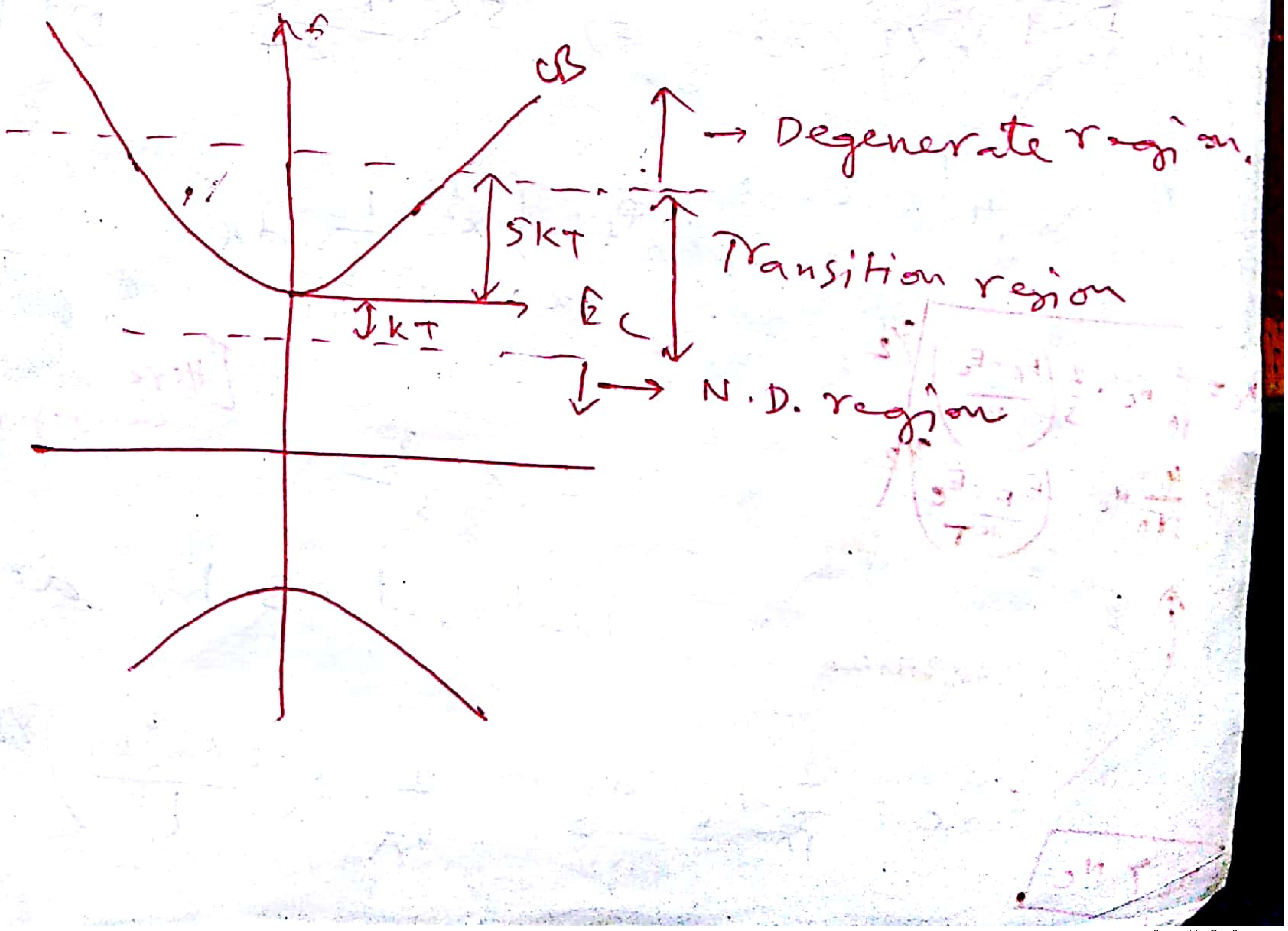
$$n_c = \frac{2}{\sqrt{\pi}} N_c\, \Phi_{\frac{1}{2}} = \frac{2}{\sqrt{\pi}} \cdot 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \cdot \frac{2}{3} \cdot \left(\frac{E_F - E_c}{KT}\right)^{3/2}$$

$$\boxed{n_c > N_c}$$

$$n_c = \frac{8\pi}{3}\left(\frac{2m_e^*}{h^2}\right)^{3/2}(E_F - E_c)^{3/2}$$

* No. of electron in the CB is independent of Temp.

Degenerate semi.	Non-degenerate semiconductor
1. Fermi level should be at least 5kT above E_c	1. fermi level should be at least kT below E_c.
2. Electron obey F.D. distribution	2) electron obeys classical distribution.
3) n_c independent on Temp.	3) n_c depend on Temp.



Non degenerate & degenerate semi conductor on the basis of holes in ~~VB~~ vallence band.

f(E) → Probability for a state of energy E to be occupied

∴ 1 − f(E) → " " " " " E to be unoccupied

∴ Carrier Concentration in vallence band
(No. of holes/volume)

$$n_h = \int_{-\infty}^{E_v} g(E)\,(1-f(E))\,dE$$

g(E) = density of states in VB (for hole)

m_h^* → effective mass of holes.

$$= \frac{4\pi(2m_h^*)^{3/2}}{h^3}(E_v-E)^{\frac{1}{2}} \; *$$

$$f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

$$\therefore \boxed{n_h = \frac{2}{\sqrt{\pi}} N_v \mathcal{F}_{\frac{1}{2}}}$$

Here $E = E_v - \frac{\hbar^2 k^2}{2m_h^*}$

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

→ effective density of states in VB.

$$\mathcal{F}_{\frac{1}{2}} = \int_0^{\infty} x^{\frac{1}{2}} \frac{1}{e^{x-\xi}+1}\,dx$$

$$x = \frac{E_v - E}{kT}$$

$$\xi = \frac{E_v - E_F}{kT}$$

Electron in

$$\Phi_{\frac{1}{2}} = \frac{\sqrt{\pi}}{2} e^{\xi} \quad \text{when } -\infty < \xi < -1 \rightarrow \text{N.D. region}$$

$$= \frac{\sqrt{\pi}}{2} \frac{1}{0.25 + e^{-\xi}} \quad \text{when } -1 < \xi < 5 \rightarrow \text{Transition Region.}$$

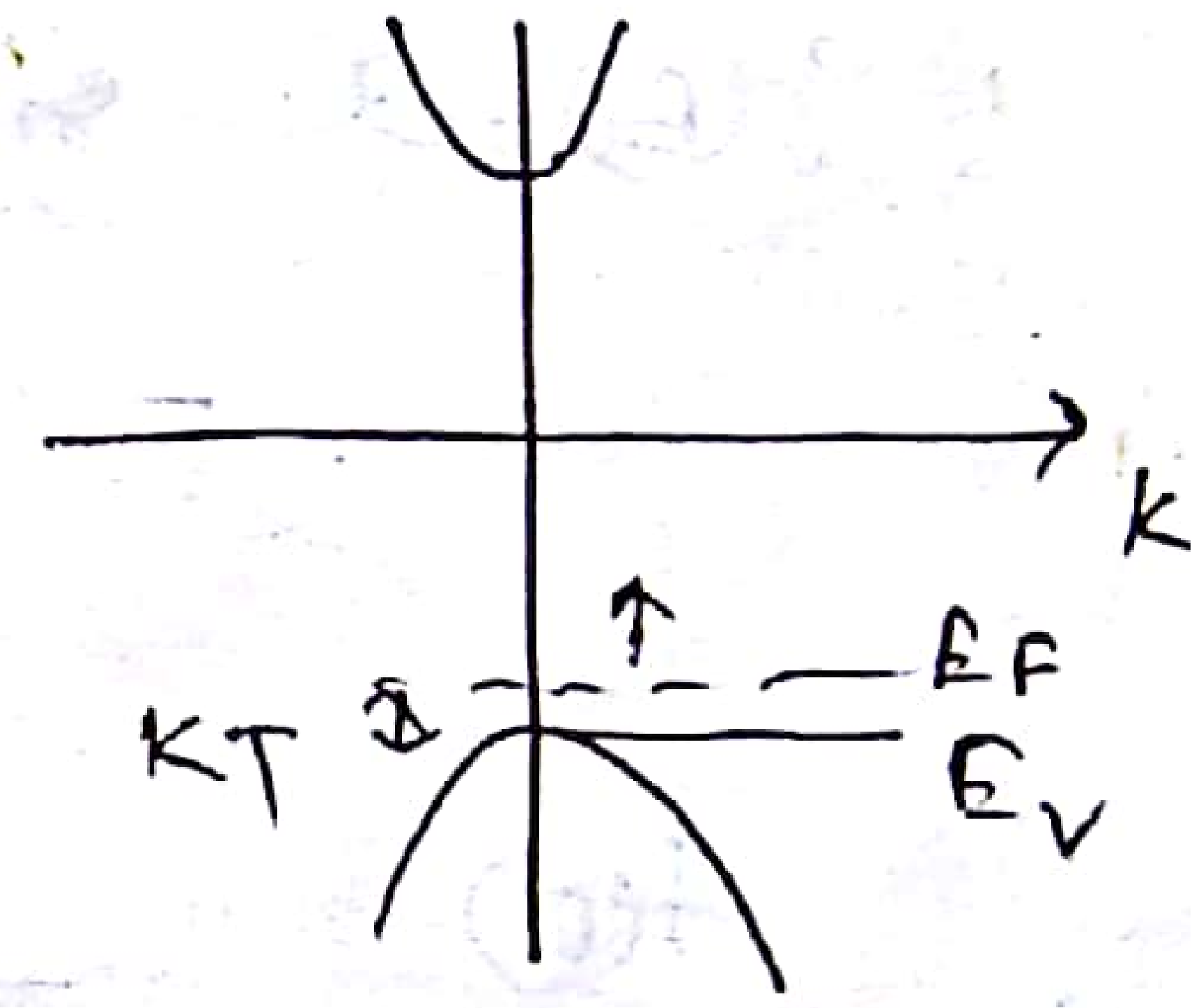
$$= \frac{2}{3} \xi^{3/2} \quad \text{when } 5 < \xi < \infty \rightarrow \text{Degenerate region.}$$

N.D. semiconductor

When $\xi < -1$, $\frac{E_V - E_F}{KT} < -1 \Rightarrow E_V < E_F - KT$

$\Rightarrow \boxed{E_F > E_V + KT}$

$\therefore E_F \rightarrow$ must be at least KT above from E_V (top of VB)



$$n_h = \frac{2}{\sqrt{\pi}} \cdot N_v \cdot \frac{\sqrt{\pi}}{2} e^{\xi}$$

$$= N_v e^{\xi}$$

$$= 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} \cdot e^{(E_V - E_F)/KT}$$

Here $n_h < N_V$ (Depend on Temp)

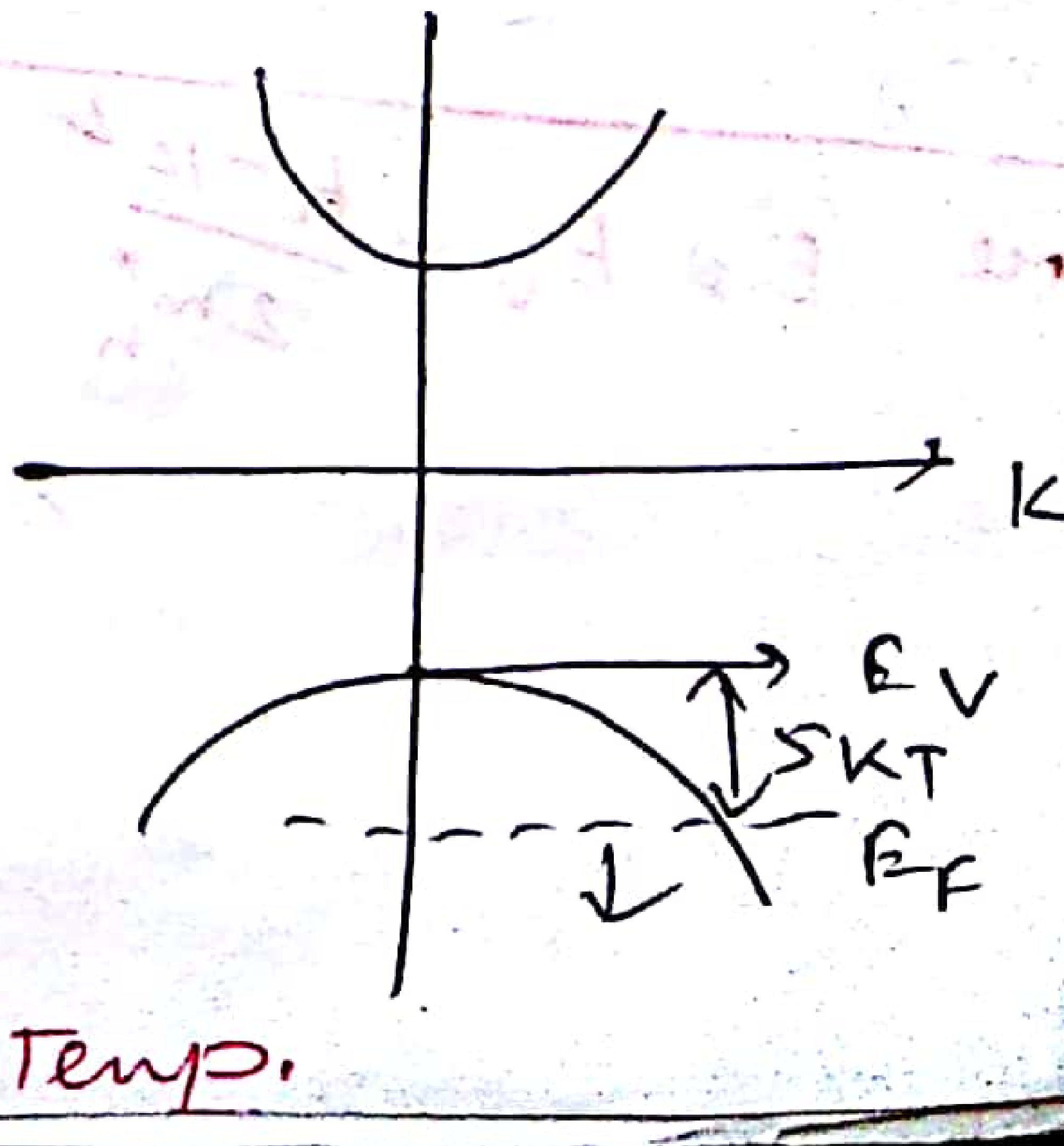
$N_V \rightarrow$ effective density of state at vallence band.

D. semiconductor

When $\xi > 5 \Rightarrow \frac{E_V - E_F}{KT} > 5 \Rightarrow E_V > E_F + 5KT$

$\Rightarrow E_F < E_V - 5KT$

Fermi level $\rightarrow$ must be at least $5KT$ below top of VB.



$$n_h = \frac{2}{\sqrt{\pi}} N_v \cdot \frac{2}{3} \xi^{3/2}$$

$$= \frac{8\pi}{3}\left(\frac{2 m_h^*}{h^2}\right)^{3/2} (E_V - E_F)^{3/2} \rightarrow \text{Independent of Temp.}$$

Here, $n_h > N_v$

Purce or Intrinsic semiconductor (Non -degener

Electron in CB is due to the transition of the carrier from VB.

Hence

No. of electron in CB = No. of holes in VB.

for Non-degenerate semi conductor

$$n_c = 2.\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_F - E_c)/kT}$$

$$n_h = 2.\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_V - E_F)/kT}$$

$$\therefore \quad n_c = n_h$$

$$\Rightarrow \left(\frac{m_e^*}{\cancel{h^2}}\right)^{3/2} e^{(E_F - E_c)/kT} = \left(\frac{m_h^*}{\cancel{h^2}}\right)^{3/2} e^{(E_V - E_F)/kT}$$

$$\Rightarrow e^{(E_F - E_c - E_V + E_F)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

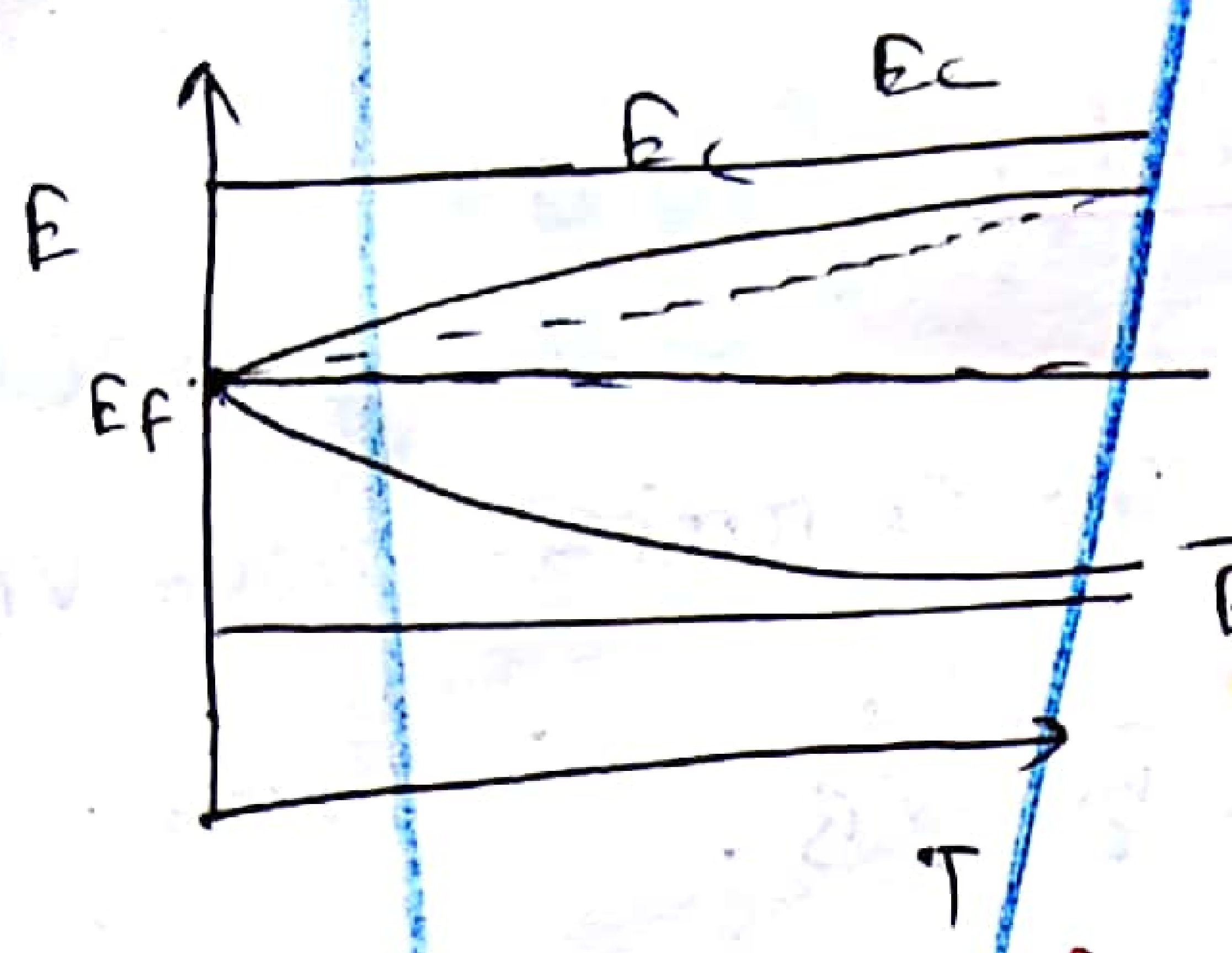
$$\Rightarrow e^{(2E_F - E_c - E_V)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$\Rightarrow 2E_F - (E_c + E_V) = kT \ln\left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$\therefore \boxed{E_F = \frac{E_c + E_V}{2} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right)}$$

Position of the fermi level in an intrinsic semi

Dependan Temp



$m_h^* > m_e^*$ (E_F increase slowly with T)

$m_h^* = m_e^*$ (E_F independant of T)

$m_h^* < m_e^*$ (E_F decrease slowly with T)

Density of electron in CB (n_c) and density of holes (n_h) in VB in terms of band gap E_g.
(Carrier Concentration in an intrinsic semiconductor)

$$E_F - E_C = \frac{E_V - E_C}{2} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$\therefore \frac{E_F - E_C}{kT} = -\frac{E_g}{2kT} + \frac{3}{4} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$\Rightarrow e^{(E_F - E_C)/kT} = e^{-E_g/2kT} \left(\frac{m_h^*}{m_e^*}\right)^{3/4}$$

2) For non-degenerate semiconductor

carrier concentration in CB

$$n_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \cdot \left(\frac{m_h^*}{m_e^*}\right)^{3/4} \cdot e^{-E_g/2kT}$$

$$= 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} \left(m_h^* m_e^*\right)^{3/4} e^{-E_g/2kT}$$

Similarly
carrier concentration in VB

$$n_h = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} \left(m_h^* m_e^*\right)^{3/4} e^{-E_g/2kT}$$

see that ⑪ (** next page)

$n_i = n_e = n_h$

$$\therefore n_i^2 = n_e n_h = 4\left(\frac{2\pi kT}{h^2}\right)^3 \left(m_h^* m_e^*\right)^{3/2} . e^{-E_g/kT}$$

$$\therefore n_i = \sqrt{n_e n_h} = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_h^* m_e^*\right)^{3/4} e^{-E_g/2kT}$$

→ Expression for carrier concentration in case of Intrinsic semicon.

Intrinsic carrier concentration for Si at room Temp. is $n_i \simeq 1.5\times 10^{10}/cc$

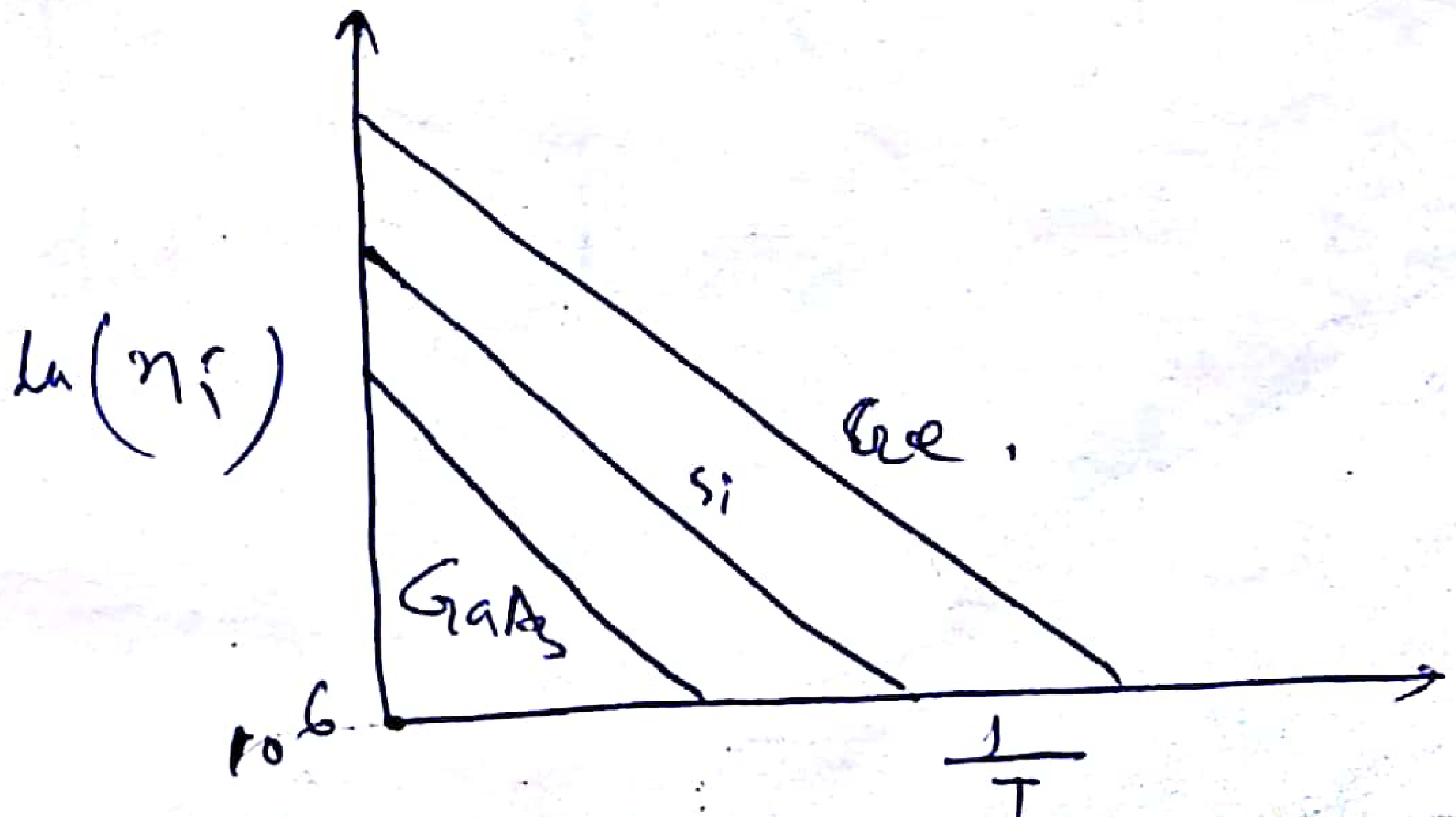
Temperature dependance of carrier concentration

$$n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_h^* m_e^*\right)^{3/4} e^{-E_g/2kT}$$

$$\ln(n_i) = \ln\left[2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_h^* m_e^*\right)^{3/4}\right] \cancel{+} - \frac{E_g}{2kT}$$

Hence $\ln(n_i)$ vs $\frac{1}{T}$ ⇒ appear almost linear.

[Here we neglect the variation ~~of~~ due to $T^{3/2}$ dependance of density of states function and the fact that E_g varies somewhat with temp.].



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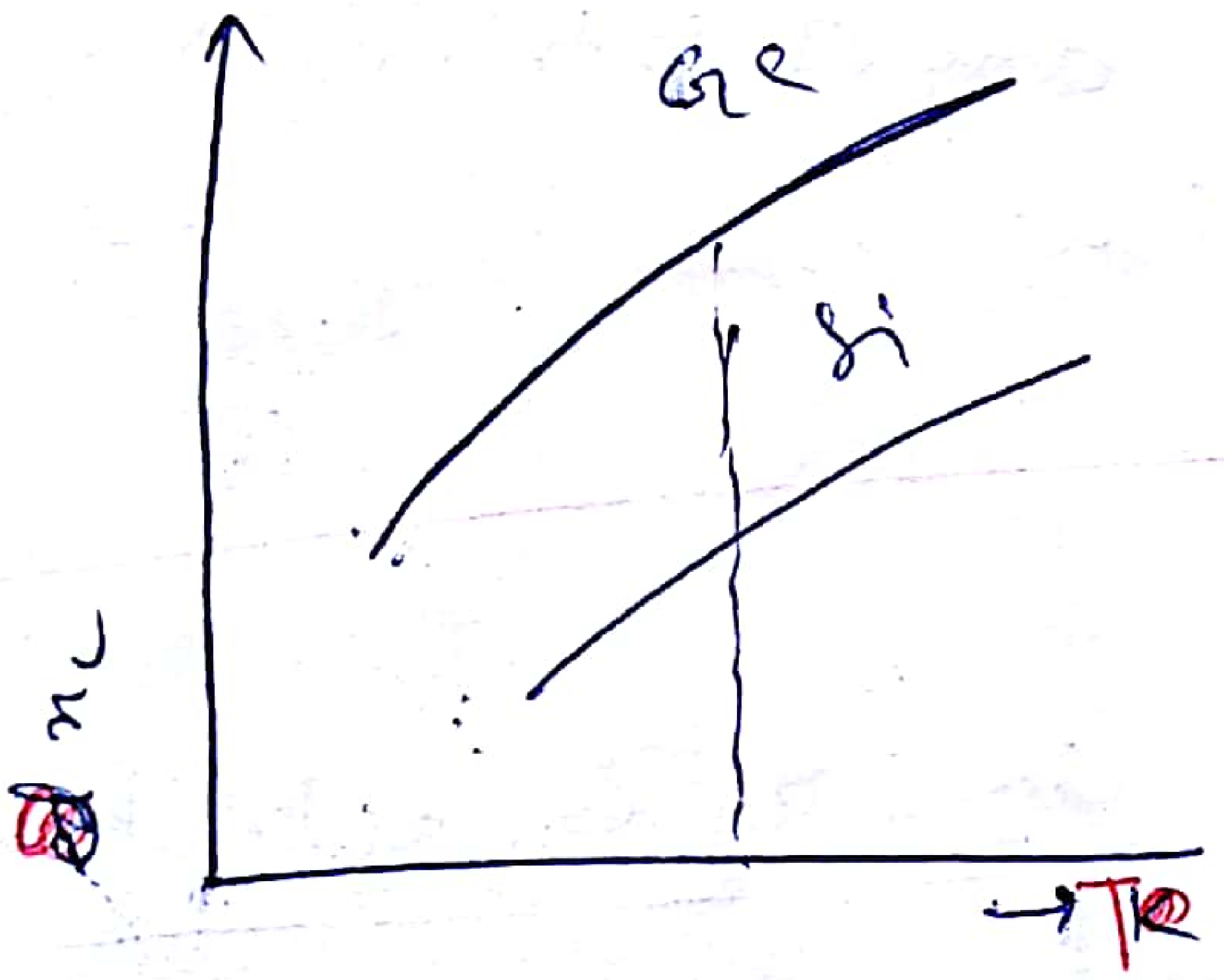
Conclusion

i) Intrinsic semiconductor density of electrons in conduction band = density of holes in VB.

ii) n_c & n_c increase exponentially as the temp. increases



The intrinsic concentration at a given temp. is

$$(n_c)_{Ge} > (n_c)_{Si}$$

because the

$$(E_g)_{Ge} < (E_g)_{Si}$$

$$\Downarrow \qquad\qquad \Vert$$

$$= 0.67\,eV \qquad 1.14\,eV.$$

ectrical conductivity

The conductivity of intrinsic semiconductor

→ Drift velocity/unit elec. field.

$$\sigma = \sigma_n + \sigma_p$$

$$= n_i e(\mu_n + \mu_p)$$

$\mu_n \rightarrow$ mobilities of electron

$\mu_p \rightarrow$ " " holes.

$$= 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_h^* m_e^*)^{3/4} \exp^{-E_g/2kT} . e(\mu_n + \mu_p)$$

Here the exponential term dominate ~~with~~ w.r.t. $T^{3/2}$ term.

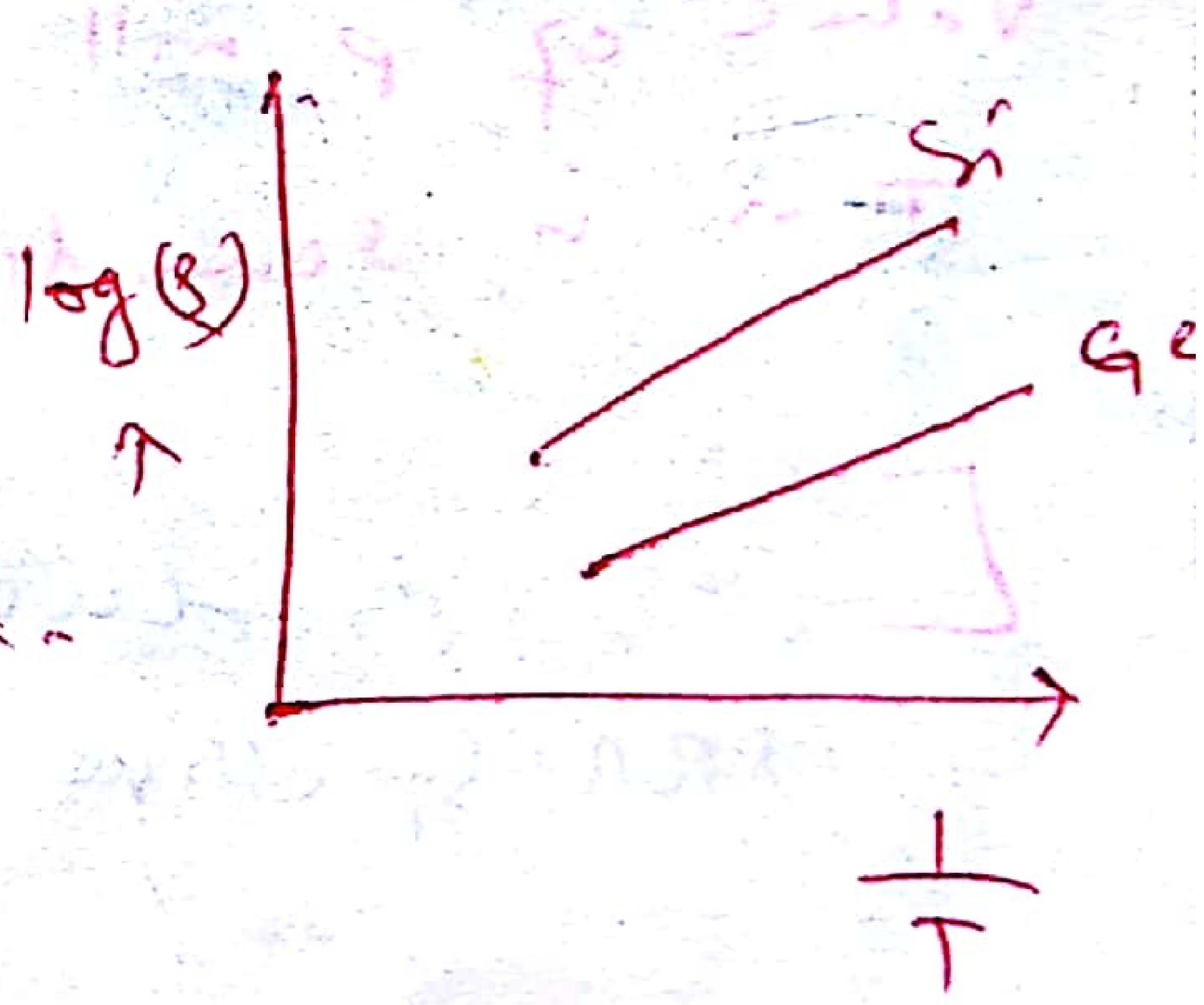
$$\therefore \sigma = (\text{constant}) \exp^{-E_g/2kT}$$

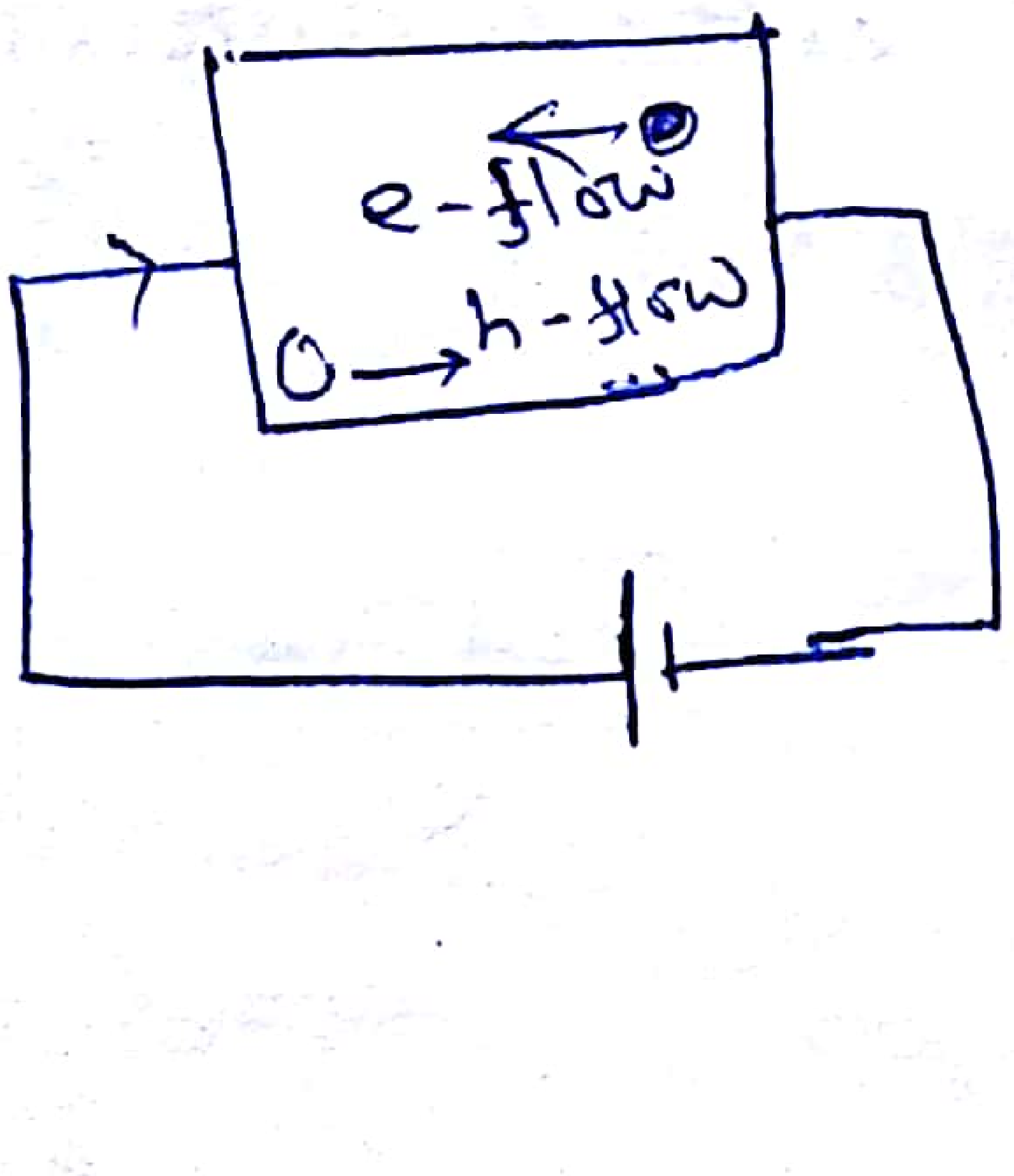
Resistivity $\rho = \frac{1}{\sigma} = \text{constant} . \exp^{E_g/2kT}$

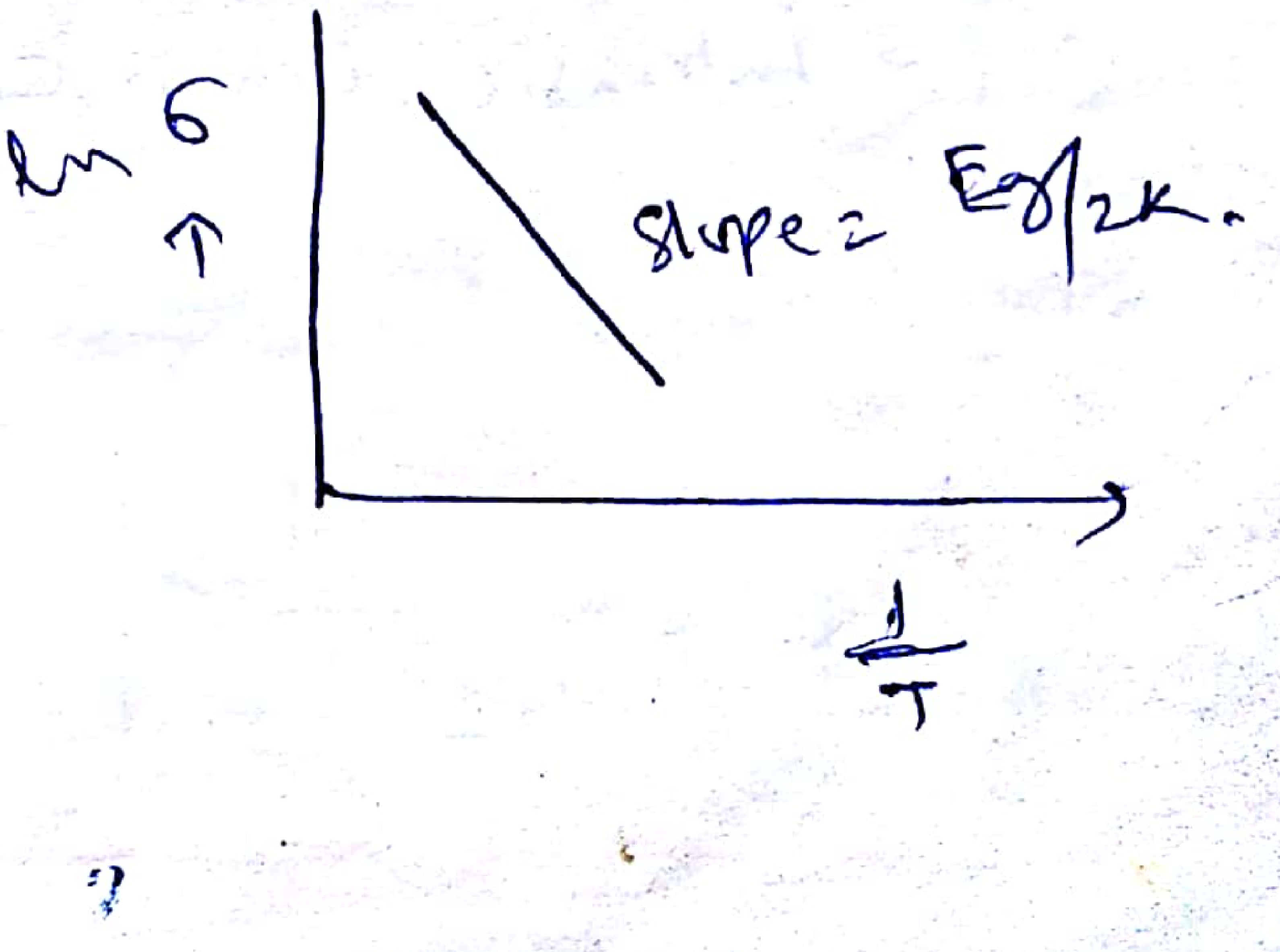
$$\therefore \log \rho = \text{constant} + \frac{E_g}{2kT}$$

$\log \rho$ vs $\frac{1}{T}$ → straight line.

From slope E_g may be calculated.







slope = $E_g/2k$.

Law of mass action

$$n_c n_h = 4\left(\frac{2\pi kT}{h^2}\right)^3 \left(m_h^* m_e^*\right)^{3/2} e^{-E_g/kT}$$

For a given temp. the products of hole & electron densities is constant & independent of Fermi level.

If n → total concentration of electron in CB
p → " " " hole in VB

then $n_e = n$
$n_h = p$

∴ $n_c n_h = np$ = constant at a constant temp.

If impurities is added to increase n, then a corresponding value of p will be decreased so that np remain constant. ~~This~~ is sometimes called law of mass action.

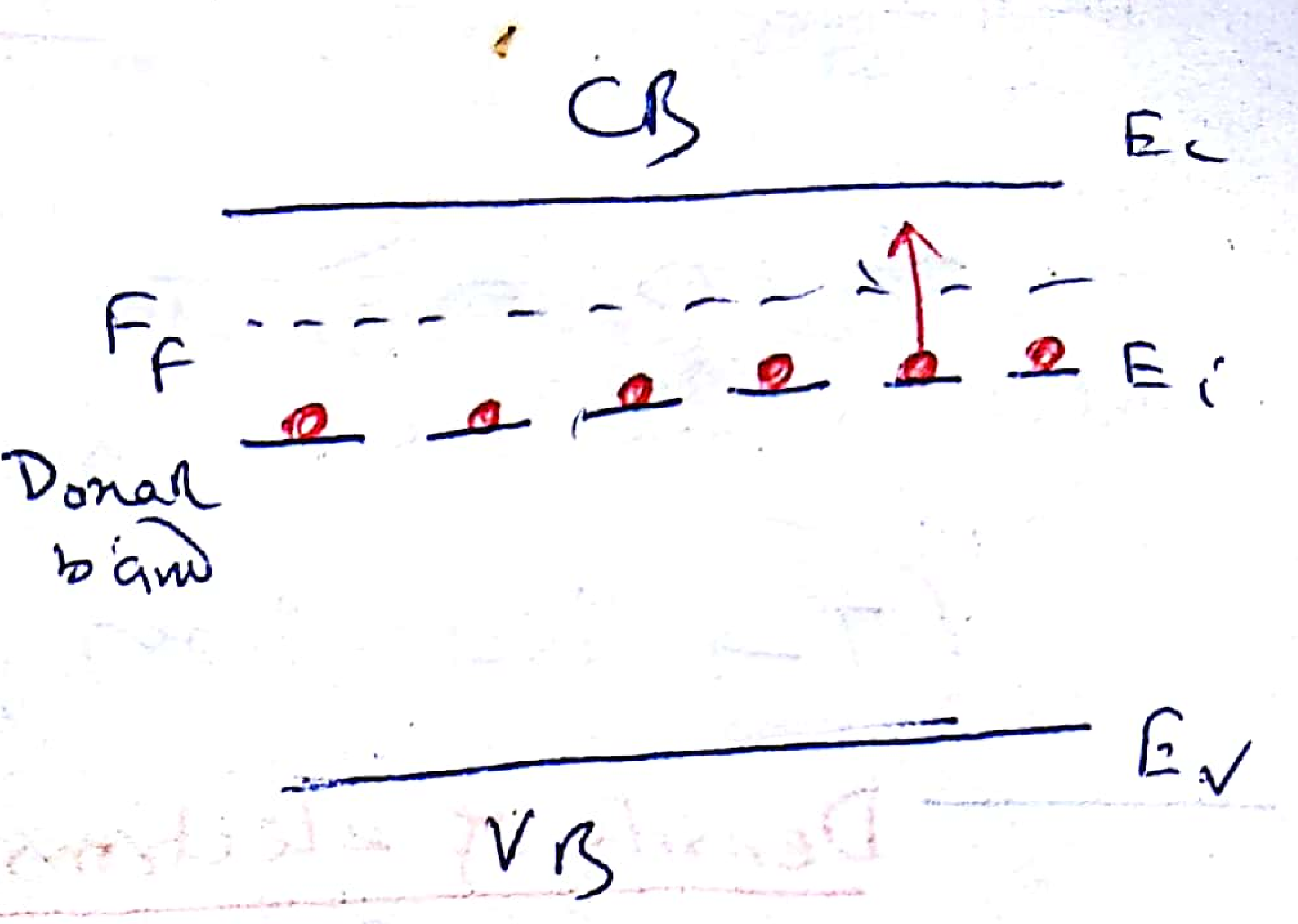
[The product np = constant for a semiconductor irrespective of it being extrinsic or intrinsic.

$$np = n_i p_i = n_i^2$$

n_i → intrinsic density of either carrier.

n-type

ype semiconductor :-



Below conduction band ~~there~~
re N_d donar level/cc
f energy E_i

$$n_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_f-E_c)/kT}$$

If E_f lies more than a few kT above Donar level. Then density of empty donors is given by

$$N_d\left(1-f(E_i)\right) \simeq N_d\, e^{(E_i-E_f)/kT}$$

Density of empty donors = density of electrons in the CB.

$$\therefore\ 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_f-E_c)/kT} = N_d\, e^{(E_i-E_f)/kT}$$

Taking logarith;

$$\ln 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} + \frac{E_f-E_c}{kT} = \ln N_d + \left(\frac{E_i-E_f}{kT}\right)$$

$$\Rightarrow \frac{2E_f}{kT} = \frac{E_c+E_i}{kT} + \ln N_d - \ln 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$\therefore\ \boxed{E_F = \frac{E_i+E_c}{2} + \frac{kT}{2}\ln\left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]}$$

Expression of Fermi energy level

E_i → energy of donar level.

At $T=0$, $E_F = \frac{E_i+E_c}{2}$.

i.e at abs. zero, the Fermi level lies exactly half way between the donor level & bottom of the CB.

$T \to$ increases, $E_F \to$ drops

Density of electrons in the CB in terms of Ionisation energy

$$E_F - E_c = \frac{E_i - E_c}{2} + \frac{kT}{2} \ln \left[\frac{\cancel{N_d}\, N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}} \right]$$

$$\therefore e^{(E_F - E_c)/kT} = e^{(E_i - E_c)/2kT} \cdot \frac{(N_d)^{1/2}}{\left[2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right]^{1/2}}$$

$$\therefore n_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \cdot \frac{(N_d)^{1/2}}{\left[2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right]^{1/2}} e^{(E_i - E_c)/2kT}$$

$$\therefore n_c = (2N_d)^{1/2} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} e^{(E_i - E_c)/2kT}$$

$$= (2N_d)^{1/2} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} e^{-\Delta E/2kT}$$

where

$\Delta E = E_c - E_i$

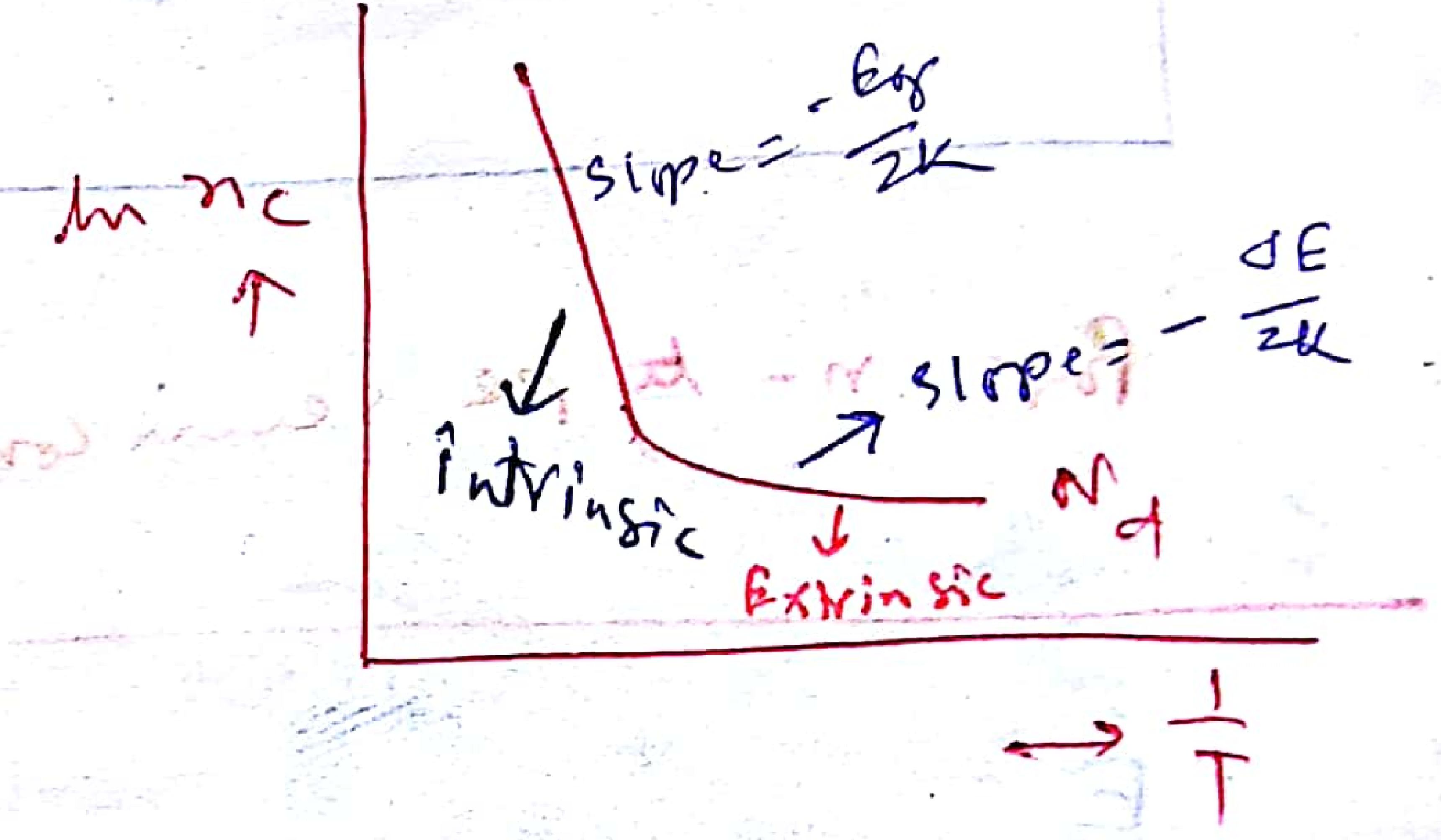
→ Ionisation energy of donors.

iclusion i) density of electron in CB ~~in~~ ∝ $\sqrt{\text{donor concentration}}$

$[n_c \propto \sqrt{N_d}$

when T increases, fermi level falls bellow the donor level & it approaches the centre of forbidden gap which makes the substance an ~~in~~ intrinsic semi conductor.

$\ln n_c$ vs $\frac{1}{T}$ graph ⇒ straight line with a slope $\left(-\Delta E/2k\right)$



When T→ sufficiently high so that, ~~@~~ the excited electron reach directly to CB from VB, then the slop change to $\left(-E_g/2k\right)$ → Intrinsic Semi.

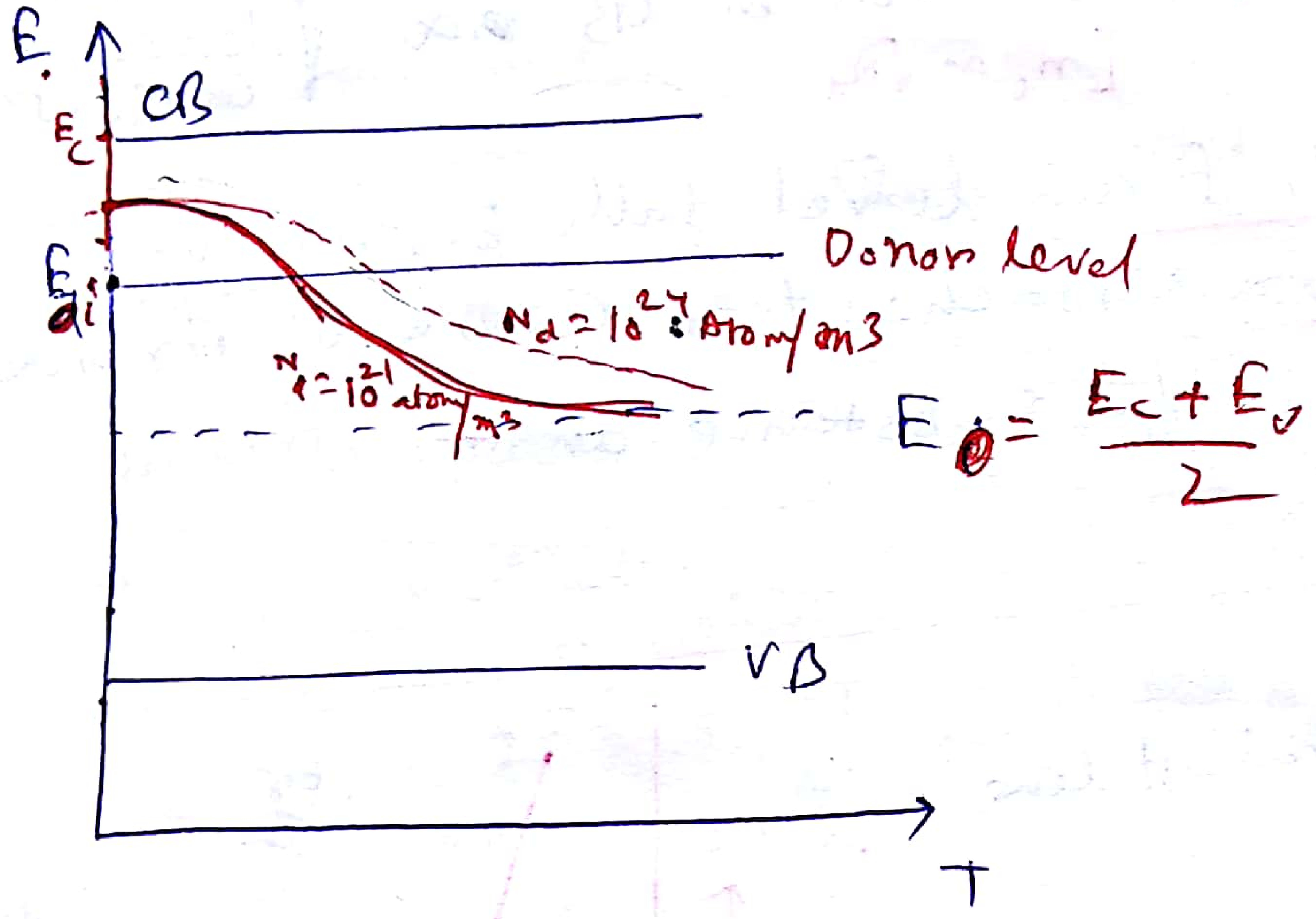
Electrical conductivity of a pure n type semiconductor

$$\sigma = e n_c \mu_n$$

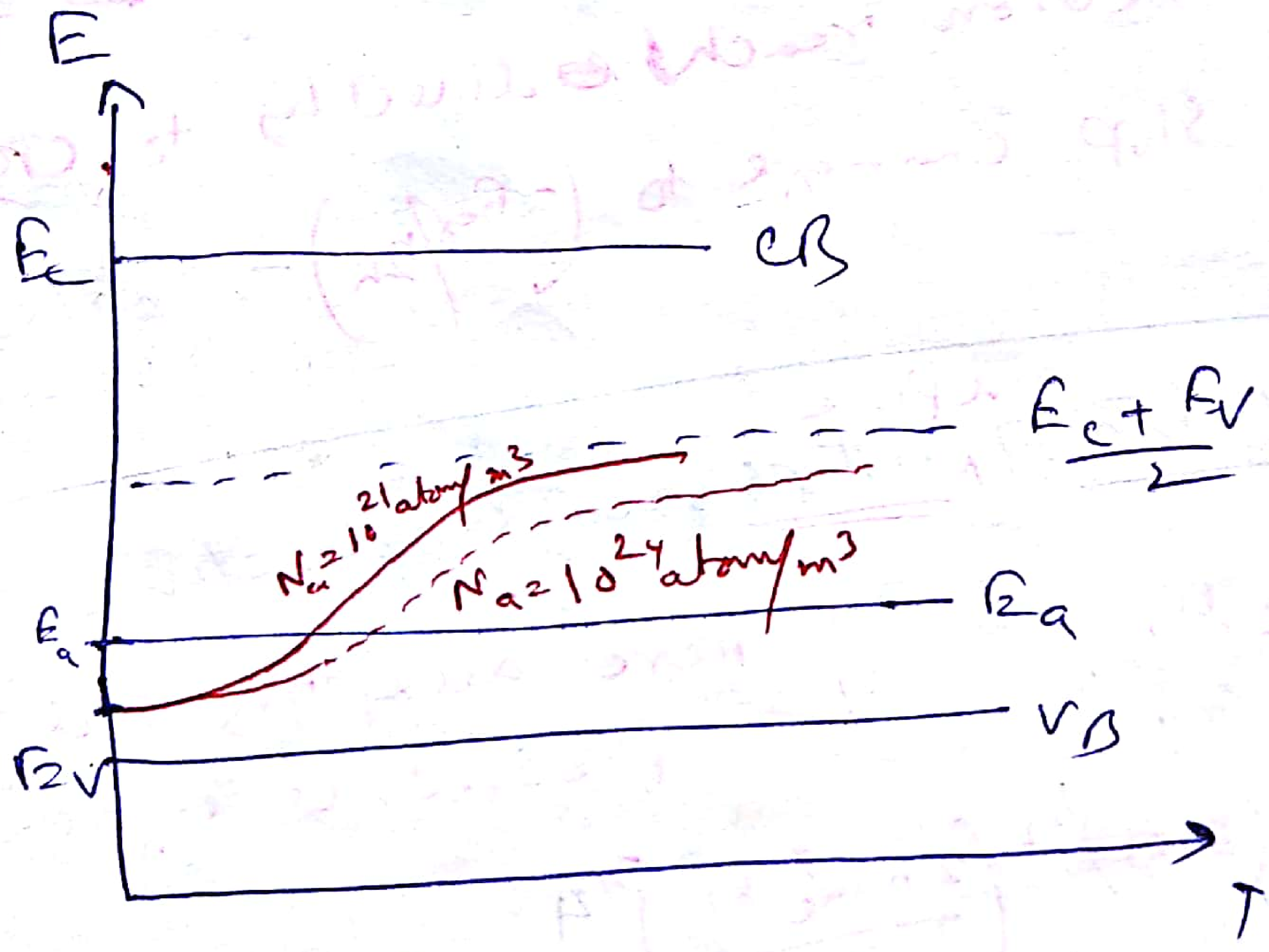
[Here acceptors are not present i.e. $\mu_h = 0$]

$$= e\mu_n \cdot (2N_d)^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} \exp\left(-\Delta E/2kT\right)$$

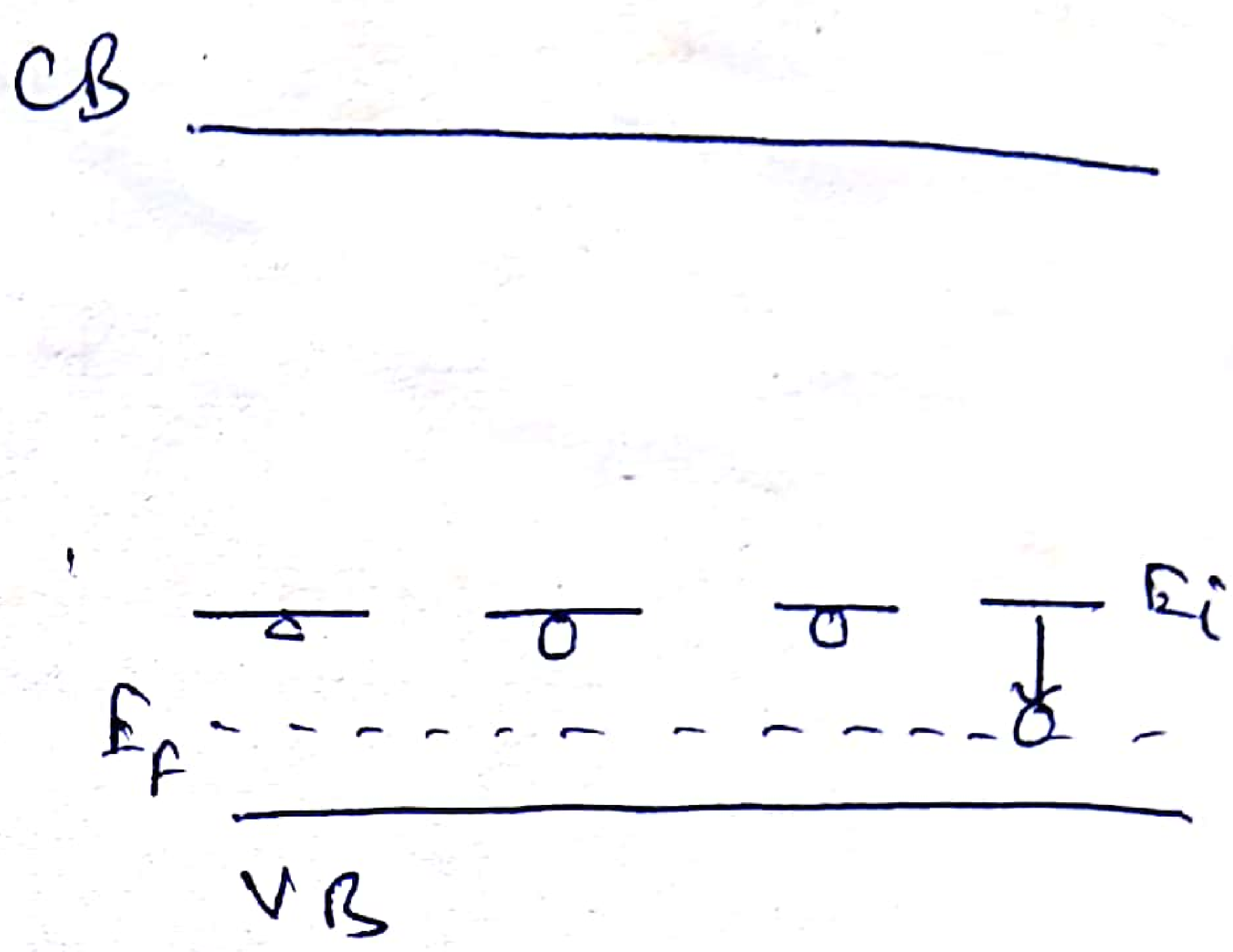
Variation of fermi level with temp.



for n-type semiconductor



p-Type semiconductor



Here acceptor levels lie above the VB.

Similarly as shown in n-type semiconductor

$$n_h = (2N_d)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} \exp\left(-\Delta E/2kT\right)$$

Here $\Delta E = E_i - E_v$

~~At~~ & $$E_F = \frac{E_i + E_v}{2} + \frac{kT}{2}\ln\left[\frac{N_d}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]$$

At $T \to 0$, $E_F = \frac{E_i + E_v}{2}$ → Fermi level lies exactly half way between the acceptor level & top of the VB.