P-1 Reliability

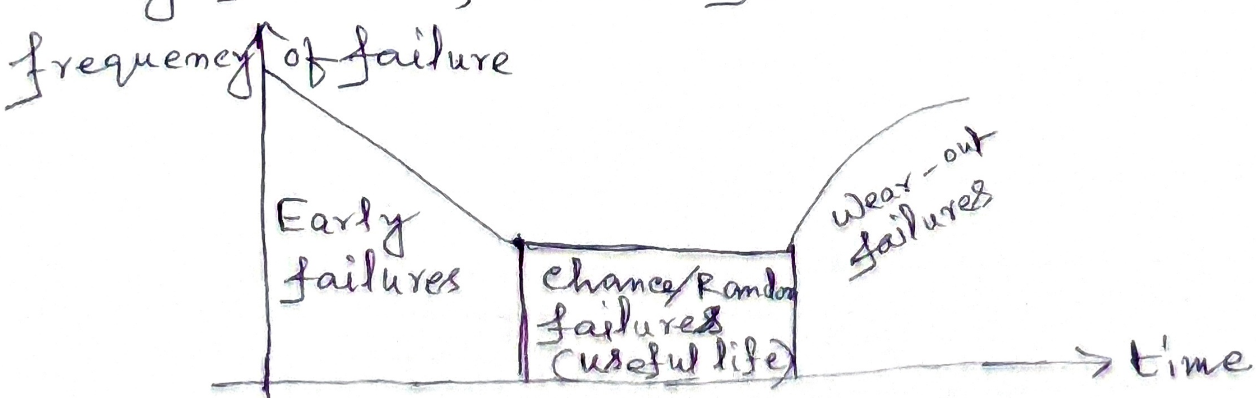
Defn The reliability is defined as the probability of a device performing its intended purpose adequately for the period of time intended under the given operating conditions.
The reliability is the probability with which the devices will not fail to perform a required operations for a certain length of time. Such a probability is known also as the 'probability of survival' (i.e., Reliability).

This defination bring into focus four important factors namely,

(a) the reliability of device is expressed as a probability.
(b) the device is required to give the intended adequate performance.
(c) the duration of adequate performance is specified. and
(d) the environmental or operating conditions are prescribed.

Life characteristic pattern:

In practice, even the best design, manufacturing and maintanance efforts of completely eliminated the occurance of failures. During the life of a system, we may experiance three different types of failures — Early failures, Random failure and wear-out failures.



Life-characteristic Curve.

Measure of Reliability: The predection of system reliability is based a number of factors such as
(i) life characteristics (ii) operating Conditions and (iii) failure distribution.

If a random sample of items are taken from the population and are put to test (or use) under a set of fixed (or given) environment/operating Conditions. Some number of sample will fail successively in time. The data so obtained represent the life length of each ~~time~~ item. The life length can be measured depending on whether the item is repairable (T.V., Radio, Computer, mobile phone etc) and non repairable (missiles, fuses etc).

For repairable items, the life can be measured by failure rate or Mean Time ~~Bet~~ between Failures (MTBF) where as for non-repairable item, the life can be measured by Mean Time To Failure (MTTF).

Failure rate: The failure rate λ is defined the number of failures in a given time interval

i.e., $$\lambda = \frac{\text{Number of failures}}{\text{Total operating hour/time}} = \frac{f}{T} \quad \text{--- (1)}$$

where λ = failure rate,
f = no. of failures during the test interval
T = Total test time.

obviously, the smaller the value of the failure rate, the higher is the reliability of the system.

Mean Time Between Failure (MTBF)

During the operating period, when the failure rate is constant, the MTBF is the reciprocal of the constant failure rate, $m = \frac{1}{\lambda}$ $\left[= \int_0^\infty R(t)\,dt, \text{ where } R(t) \text{ is the reliability}\right]$

MTBF is also referred ~~as~~ to as the average time of satisfactory operation of the system. In this case, the ~~large~~ larger the MTBF, higher is the reliability of the system.

Remark: The failure rate and MTBF are used when the items are repairable.

Mean Time To Failure (MTTF):

If we have a life-test information on n-item item with failure items $t_1, t_2, \ldots, t_n$ then MTTF is defined as $MTTF = \frac{1}{n}\sum_{i=1}^{n} t_i$

Remark: MTTF is used when the items are non-repairable.

Derivation of Reliability function:

Let N_f = a fixed no. of components/items at the begining of the test, which are put under test repeatedly.

N_s = Number of components/items failed during a specific time, t

N_s = Number of items still surviving after the specific time, t.

As the time of test increases, N_f will increase and N_s will decrease at the same rate and as a result

$N_o = N_s + N_f$ will remain constant. Thus, the reliability or the probability of survival is defined as

$$R(t) = \frac{N_s}{N_o} = \frac{N_s}{N_s + N_f} \quad \text{——— (1)}$$

The probability of failure at any time t is

$$Q(t) = \frac{N_f}{N_o} = \frac{N_f}{N_s + N_f} \quad \text{——— (2)}$$

Thus, at time t, $R(t) + Q(t) = 1$.

Derivations: (i) Since R(t) and Q(t) are mutually exclusive

$$R(t) = 1 - Q(t) = 1 - \frac{N_f}{N_o}$$

$$\therefore \frac{dR(t)}{dt} = \frac{d}{dt}\left(1 - \frac{N_f}{N_o}\right) = -\frac{1}{N_o}\frac{dN_f}{dt}$$

$$\therefore \frac{dN_f}{dt} = -N_o \frac{dR(t)}{dt} \quad \text{——— (3)}$$

This gives the rate at which the components/items fail at any time, t

As $N_f = N_o - N_s$

$$\frac{dN_f}{dt} = \frac{dN_o}{dt} - \frac{dN_s}{dt} = 0 - \frac{dN_s}{dt} = -\frac{dN_s}{dt}$$

Thus $\frac{dN_f}{dt}$ can be interpreted as the number of components/items failing in the time interval, dt between the time t and t+dt, which is equivalent to the rate at which the component population (still in test/life) at time t is failing (in magnitude). Thus the rate of components failed during the time dt is equal to the negative rate of components survived during dt.

(ii) from (1), $\frac{dN_f}{dt} = -N_0 \frac{dR(t)}{dt}$

or, $\frac{1}{N_s}\frac{dN_f}{dt} = -\frac{N_0}{N_s}\frac{dR(t)}{dt} = -\frac{1}{\left(\frac{N_s}{N_0}\right)}\frac{dR(t)}{dt} = -\frac{1}{R(t)}\frac{dR(t)}{dt}$

as, $\frac{N_s}{N_0} = R(t)$

or, $\lambda(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt}$, where $\lambda(t) = \frac{1}{N_s}\frac{dN_f}{dt}$, the failure rate of the component —— (4)

Because, at time t, N_f is the no. of failed components and we have still N_s components intest. So, $\frac{dN_f}{dt}$ components will ~~failled~~ fail out of the N_s components. Thus we get the failure rate $\lambda(t)$ at time t per one component which is given by (4). Integrating;

$$\int_0^t \lambda(t)\,dt = -\int_{t=0}^{t} \frac{1}{R(t)}\frac{dR(t)}{dt}\,dt$$

$$= -\int_{t=0}^{t} \frac{1}{R(t)}\,dR(t) = -\log R(t)$$

$$\therefore \log R(t) = -\int_0^t \lambda(t)\,dt$$

or, $R(t) = \exp\left[-\int_0^t \lambda(t)\,dt\right]$ —— (5)

which gives a general formula for computing reliability.

Particular Case: If $\lambda(t)$ is constant over the time, i.e., $\lambda(t) = \lambda$ (say), then $R(t) = \exp\left[-\int_0^t \lambda\, dt\right] = e^{-\lambda t} = e^{-t/m}$

where m = MTBF.

Properties of Reliability function:

(i) Since the reliability is probability function,

$$0 \leq R(t) \leq 1$$

(ii) $R(0) = 1$ and $\lim_{t \to \infty} R(t) = 0$

(iii) $R(t)$ is a decreasing function of time t.

Ex: A device with 1000 hrs useful life at a constant failure rate 0.0001 per hr in a given experiment. What is the reliability for 10 hrs. operation of this device. Find the value of MTBF.

Ans: Reliability $= R(t) = e^{-\lambda t}$

λ = failure rate = 0.0001 per hr.

t = ~~of~~ operation time = 10 hrs

$$\text{Reliability} = R(10) = e^{-\lambda \cdot 10} = e^{-0.0001 \times 10} = e^{-0.001}$$

$$\simeq 0.999$$

$$\text{MTBF} = \frac{1}{\lambda} = \frac{1}{0.0001} = 10{,}000 \text{ hrs.}$$

Ex: For an equipment, the reliability per 100 hrs operation has been estimated to be 0.999. What is the function rate of the equipment? What is MTBF?

Ans: Reliability, $R(t) = 0.999$ (given) for 100 hrs operation. $\therefore t = 100$ hrs.

$\therefore R(t) = e^{-\lambda t} = 0.999$ when $t = 100$

$\Rightarrow 1 - \lambda t \simeq 0.999$ where $t = 100$.

or, $1 - 100\lambda = 0.999$

$\Rightarrow \lambda = 0.00001$

$\therefore$ Failure rate $= \lambda = 10^{-5}$ per hr

$\therefore$ MTBF $= \frac{1}{\lambda} = 10^{5}$ hrs.

Failure rate $\lambda(t)$

We have $\lambda = \frac{1}{N_s}\frac{dN_s}{dt} = -\frac{N_0}{N_s}\frac{dR(t)}{dt}$, if λ is constant, then $\frac{1}{N_s}\frac{dN_f}{dt}$ is constant. Again $\frac{dN_f}{dt}$ will remain constant if the number of failed components increases linearly with time t from the begining of the test.

So, in that case, $\frac{dN_f}{dt} = \frac{N_f}{t}$

[as N_f increases linearly with time t, then $N_f = at+b$ (say)

Taking $b = 0$, $N_f = at$, $\frac{dN_f}{dt} = a = \frac{N_f}{t}$]

$$\therefore \lambda = \frac{1}{N_s}\frac{dN_f}{dt} = \frac{1}{N_s}\left(\frac{N_f}{t}\right) = \frac{1}{N_0}\frac{N_f}{t}$$

When failed components are substituted immidiately after failure. i.e, $N_s = N_0$ for all time.

Therefore in that case, to calculate the failure rate, we need to count the no. of failures N_f and the hour of operation t.

We have, $\frac{1}{N_s}\frac{dN_f}{dt} = -\frac{N_0}{N_s}\frac{dR(t)}{dt}$

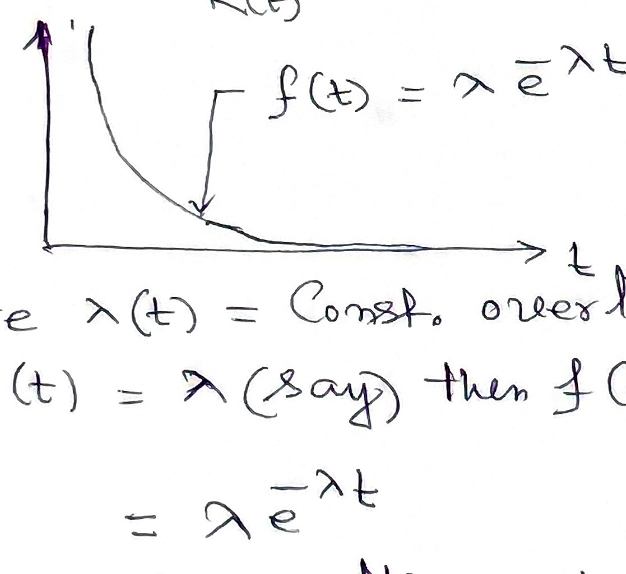
$$\therefore \frac{dR(t)}{dt} = -\frac{1}{N_0}\frac{dN_f}{dt}.$$

Where $\frac{1}{N_o}\frac{dN_f}{dt}$ is plotted against t, we obtain the distribution function of failures at time t per component basis. Failure frequency curve per component. This curve is known as 'failure density function' or 'distribution curve'.

i.e., $f(t) = \frac{1}{N_o}\frac{dN_f}{dt} = -\frac{dR(t)}{dt}$

$\therefore \lambda = -\frac{N_o}{N_s}\frac{dR(t)}{dt} = -\frac{1}{\left(\frac{N_s}{N_o}\right)}\frac{dR(t)}{dt} = -\frac{1}{R(t)}\frac{dR(t)}{dt}$

$= \frac{f(t)}{R(t)}$, where $f(t) = -\frac{dR(t)}{dt}$.



Where $\lambda(t)$ = Const. over time t,

i.e., $\lambda(t) = \lambda$ (say) then $f(t) = -\frac{dR(t)}{dt} = -\frac{d}{dt}\left(e^{-\lambda t}\right)$

$= \lambda e^{-\lambda t}$

Moreover, $Q(t) = \frac{N_f}{N_o}$ $\therefore \frac{dQ(t)}{dt} = \frac{1}{N_o}\frac{dN_f}{dt} = f(t)$

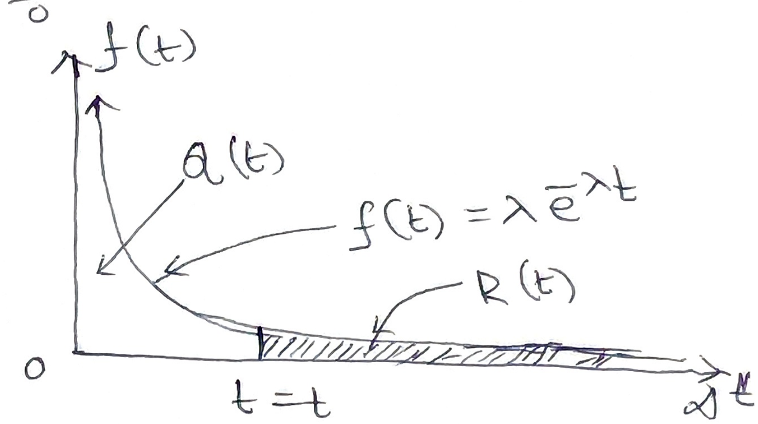
$\therefore Q(t) = \int_0^t f(t)\,dt$.

This means that probability of failure Q(t) at time t is equivalent to the area under the density curve taken from t = 0 to t = t.

Then $R(t) = 1 - Q(t) = 1 - \int_0^t f(t)\,dt = \int_0^\infty f(t)\,dt - \int_0^t f(t)\,dt$

$$= \int_t^\infty f(t)\,dt \quad \text{as} \quad \int_0^\infty f(t)\,dt = 1.$$

Thus we have,



Hazard Rate Z(t)

The Hazrad rate is defined as the limet t of the failure rate as the interval length approaches to zero. $\therefore$ $Z(t) = \lim_{h \to 0} \frac{R(t) - R(t+h)}{h\,R(t)}$

$$= \frac{1}{R(t)}\left[-\frac{dR(t)}{dt}\right] = \frac{f(t)}{R(t)}$$

as $f(t) = -\frac{dR(t)}{dt}$

Relationship between Z(t) and R(t).

$$Z(t) = \frac{f(t)}{R(t)} = \frac{1}{R(t)}\frac{dQ(t)}{dt}$$

$$\therefore \int_0^t Z(t)\,dt = \int_{t=0}^{t} \frac{1}{R(t)} \frac{dQ(t)}{dt}\,dt = \int_{t=0}^{t} \frac{1}{R(t)}\,dQ(t)$$

$$= \int_{t=0}^{t} \frac{1}{1-Q(t)}\,dQ(t) = -\left[\log(1-Q(t))\right]_{t=0}^{t} = -\left[\log R(t)\right]_{t=0}^{t}$$

$$= -\log R(t) \Rightarrow R(t) = \exp\left(-\int_0^t Z(t)\,dt\right)$$

$$Q(t) = 1 - R(t) = 1 - \exp\left\{-\int_0^t Z(t)\,dt\right\}$$

$$f(t) = -\frac{dR(t)}{dt} = Z(t)\exp\left\{-\int_0^t Z(t)\,dt\right\}$$

Which is the required relation.

Generally, i) to determine the reliability of a system, the system is broken down to subsystem and elements whose individual reliability factors can be estimated or determined.

(ii) Depending upon the manner in which these ~~system~~ sub-systems or elements are connected, the Combinat~~orid~~ -orial which rules of probability are applied.

Then, basics steps for this are:

1. The elements and sub-systems which constitute the given system and whose individual reliability factors can be estimated are identified.
2. The logical manner or configuration in which these elements are connected to form the system is represented by a block diagram or a circuit diagram.
3. The condition for successful operation of the system is then determined i, it may be decided as to how the units should function.
4. Finally, the combinatorial rules of probability theory (i.e, addition rule, multiplication rule, etc.) are applied to arrive at the system reliability factor.

Reliability of series systems:- In series system, a large number of components of the system are connected in series. A system comprising of n components in series may represented as

$$\rightarrow \boxed{1} \rightarrow \boxed{2} \rightarrow \boxed{3} - - - \rightarrow \boxed{n} \rightarrow$$

In series system, if any one of the components fails, the system fails. In other words, if the system is to operate successfully, then each component connected in series should operate.

Let the successful operation of these individual components be represented by $X_1, X_2, \ldots, X_n$ and their representive probability by $P(X_1), P(X_2), \ldots, P(X_n)$.

Hence, if the components are not independent of one another, then system reliability is

$$P(S) = P(X_1 \text{ and } X_2 \text{ and } X_3 \text{ and } \ldots \text{ and } X_n)$$

$$= P(X_1)P(X_2/X_1)P(X_3/X_1X_2)\ldots P(X_n/X_1 \text{ and } X_2 \ldots \text{ and } X_{n-1})$$

where S is the successive operation of the system.

If the system successful operation of each component is independent of the successful operation of the remaining units, then $P(S) = P(X_1).P(X_2)\ldots P(X_n)$.

Let R_i be the reliability of the ith component connected in series in the system and R_s be the reliability of the system. Let there are n components in the system

Then $R_s = R_1 R_2 R_3 \ldots R_n = \prod_{i=1}^{n} R_i$

If $R_1 = R_2 = \ldots = R_n = R$ (say).

$R_s = R.R.R \ldots R = R^n = (1-Q)^n$

where Q is the probability of failure of each component. When each component has an exponential time-to-failure density, then

$R_s(t) = R_1(t) R_2(t) \ldots R_n(t) = e^{-\lambda_1 t} . e^{-\lambda_2 t} \ldots, e^{-\lambda_n t}$

where λ_i is the failure rate of ith component

$$= \exp\left[-\sum_{i=1}^{n}\lambda_i t\right] = e^{-\lambda_s t}, \text{ where } \lambda_s = \sum_{i=1}^{n}\lambda_i .$$

The mean time between failures (MTBF) for the system having n components connected in series is $m_s = \int_0^\infty R_s(t)\,dt = \int_0^\infty \exp[-(\lambda_1+\lambda_2+\cdots+\lambda_n)t]\,dt$

$$= \frac{1}{\lambda_1+\lambda_2+\cdots+\lambda_n} = \frac{1}{\lambda_s}, \text{ where } \lambda_s = \lambda_1+\lambda_2+\cdots+\lambda_n$$

If $\lambda_1 = \lambda_2 = \cdots = \lambda_n$ then $\lambda_s = n\lambda$,

$\therefore m_s = \frac{1}{n\lambda}$; If $n=1$, $m_s = \frac{1}{\lambda}$.

Ex: An Electric circuit consists of 5 silicon transistors 3 silicon diodes, 10 resistors and 2 capacitors in series configuration. The hourly failure rates are transistors: $\lambda_t = 4\times10^{-5}$; diods: $\lambda_d = 3\times10^{-5}$; resistors: $\lambda_r = 2\times10^{-4}$; capacitor: $\lambda_c = 2\times10^{-4}$. Calculate the reliability of the circuit for 10 hrs when the components follow exponential distribution.

Ans: Let R_s be the reliability of the circuit.
λ_s be the failure rate of the circuit.

$R_s(t) = e^{-\lambda_s t}$; where $\lambda_s = \sum\lambda_i = 5\lambda_t + 3\lambda_d + 10\lambda_r + 2\lambda_c = 20\times10^{-5} + 3\times10^{-5} + 20\times10^{-4} + 4\times10^{-4}$

$= 0.00269$ per hr.

$R_s(t) = e^{-\lambda_s t} = e^{-0.00269t}$

Reliability of the circuit for 10 hrs.

$R_s(10) = e^{-0.00269\times10} = e^{-0.0269} \simeq .9735$.

Physical meaning: The circuit is expected to operate

9735 times without failure and ~~wd~~ would fail 265 times out of 10,000 operations of 10 hrs each.

MTBF (mean time between these failures) in this case is, $m_s = \frac{1}{\lambda_s} = \frac{1}{0.00269} = 371.75$ hrs.

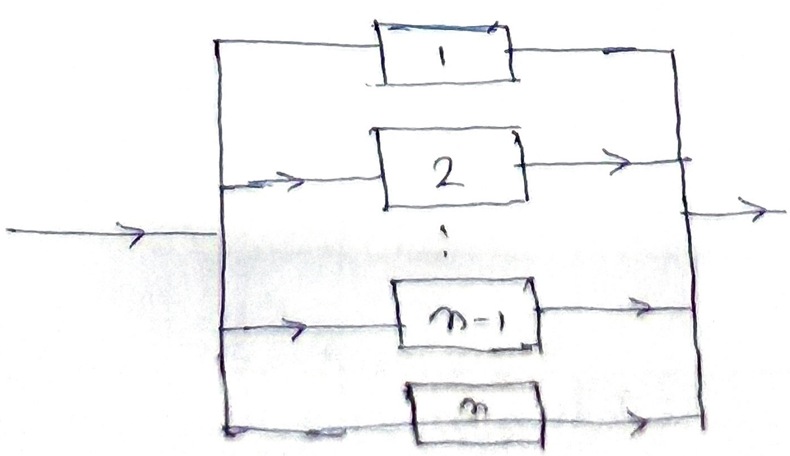
This means that the circuit is expected to operate without failure for 372 hrs.

Ex: A system is connected in series consists of 500 transistors, 1500 resistors, 500 capacitors, 1000 diods.
Failure rates of these components are:

For transistors : $\lambda_t = 0.7 \times 10^{-7}$ per hour.
" Diods : $\lambda_d = 0.3 \times 10^{-6}$ " "
" resistors : $\lambda_r = 0.1 \times 10^{-6}$ " "
" capacitors : $\lambda_c = 0.2 \times 10^{-6}$ " "

What is the failure rate of the system. What is the reliability? What is the MTBF. Furnish their physical meaning.

Reliability of parallel system:



$$P(\bar{x}) = P(\bar{x}_1 \text{ and } \bar{x}_2 \text{ and } \bar{x}_3 \text{ and} -- \text{ and } \bar{x}_n)$$
$$= P(\bar{x}_1)\, P(\bar{x}_2/\bar{x}_1)\, P(\bar{x}_3/\bar{x}_1\bar{x}_2) - - P(\bar{x}_n/\bar{x}_1 \text{ and } \bar{x}_2 - \text{ and } \bar{x}_{n-1})$$

Independent, $P(S) = P(\bar{x}_1) P(\bar{x}_2) \cdots P(\bar{x}_n)$

$\therefore P(S) = 1 - P(\bar{S}) = 1 - P(\bar{x}_1) P(\bar{x}_2) \cdots P(\bar{x}_n)$

$= 1 - [1 - P(x_1)][1 - P(x_2)] \cdots [1 - P(x_n)]$

$= 1 - \prod_{i=1}^{n} [1 - P(x_i)]$

Hence the reliability of a system comprising of n components in parallel is

$R_s(t) = 1 - \prod_{i=1}^{n} Q_i(t)$, $Q_i(t) = 1 - P(x_i)$

= the unreliability of the ith components.

$= 1 - \prod_{i=1}^{n} [1 - R_i(t)]$

If the failure rate of each unit is exponential time-to-failure distribution then

$R_i(t) = e^{-\lambda_i t}$ where λ_i is the failure rate of the ith unit/component.

$\therefore R_s(t) = 1 - \prod_{i=1}^{n} [1 - R_i(t)] = 1 - \prod_{i=1}^{n} \{1 - e^{-\lambda_i t}\}$

For identical units, $\lambda_1 = \lambda_2 = \cdots = \lambda_n = \lambda$ (say)

Then $R_s(t) = 1 - (1 - e^{-\lambda t})^n$.

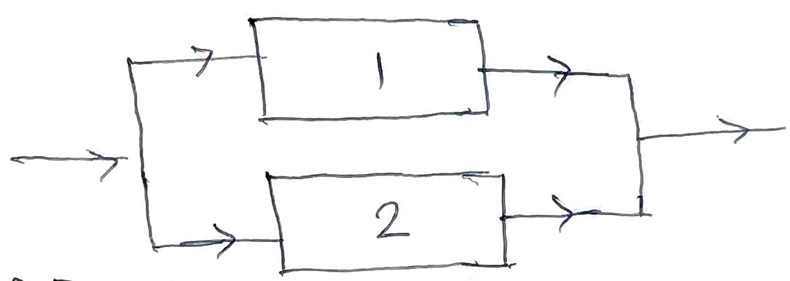
(MTBF) Mean Time Between Failures: The MTBF of a system having two components connected in parallel, can be obtained by integrating the reliability function over the range of t from 0 to ∞.

$$m_s = \int_0^\infty R_s(t)\,dt = \int_0^\infty [1-(1-R_1(t))(1-R_2(t))]\,dt \quad [\text{for } n=2]$$

$$= \int_0^\infty [1-(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t})]\,dt.$$

$$= \int_0^\infty [e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}]\,dt = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1+\lambda_2}$$

If these two components are identical, then $\lambda_1 = \lambda_2 = \lambda$ (say) and $m_s = \frac{1}{\lambda} + \frac{1}{\lambda} - \frac{1}{2\lambda} = \frac{1.5}{\lambda}$.



ie, MTBF of a parallel redundant system containing two components of equal failure rate is 1.5 times the MTBF of a single components.

Ex: If failure of an electronic subsystem is 0.0005 failure/hr. If an operational period of 500 hrs with probability of success $P(S) = 0.95$ is desired. What level of parallel redundancy is needed.

Ans: In this case, given that $\lambda = 0.0005$ failure/hr

R_s = Reliability of the system $= P(S) = 0.95$.

Let n = no. of components connected in parallel.

Now we have to find the value of n.

For a system with n components connected in parallel,
$R_s = 1 - (1-R)^n$, R be the reliability of each component

$\therefore R(t) = e^{-\lambda t} = e^{-0.0005 \times 500} = e^{-0.25}$

$\therefore .95 = 1 - (1 - e^{-0.25})^n \Rightarrow (1 - e^{-0.25})^n = 1 - .95 = .05,$

$$\therefore n = \frac{\log(.05)}{\log(1 - e^{-0.25})} = \frac{13010}{6548} \approx 2$$

Hence the level of parallel redundancy is 2.

Ex: How many identical components, each of which is 90% reliable over a period of 50 hrs be used to obtain 99.99% parallel redundancy system over 50 hrs. If we want to obtain the same system reliability over a period of 100 hrs how many component should be added.

Ans: Let n be the level of parallel redundancy.
Given that, R = reliability of each component
$= 90\% = .9.$

R_s = reliability of the system $= 99.99\% = 0.9999.$

We have, $R_s(t) = 1 - (1 - R(t))^n$

$\Rightarrow .9999 = 1 - (1 - .9)^n \Rightarrow (.1)^n = .001$ or $n = 4.$

2nd part: In this case R = reliability of each component
$= .9$ per 50 hrs.

But R_s = Reliability of the System = $\cdot 9999$ per 100 hrs.
As the reliability of the system is given for 100 hrs, we first evaluate the reliability of each component per 100 hrs.

For each component, we have $R = e^{-\lambda t}$

or, $\cdot 9 = e^{-\lambda \times 50} = e^{-50\lambda} \approx 1 - 50\lambda$.

or, $50\lambda = \cdot 1 \Rightarrow \lambda = 0.002$.

Let R' = reliability of each component per 100 hrs

$= e^{-\lambda t}$ for $t = 100$

$= e^{-0.002 \times 100} = e^{-0.2} \approx \cdot 81$

Now, we have, $R_s = (1 - (1 - R'))^n$

where n is level of parallel redun.

or, $\cdot 9999 = 1 - (1 - \cdot 81)^n \Rightarrow (\cdot 19)^n = \cdot 0001$

or, $n \approx 3$.

Hence, three identical parallel redundancy is required.

Ex. Two items have an MTBF of 50 hrs each. What is the MTBF of a parallel system of these two units.

Ans. m_s = MTBF of the parallel System

$= \frac{3}{2} \cdot \frac{1}{\lambda} = \frac{3}{2} \times 50$ hrs $= 75$ hrs.