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End Semester Examination of Semester–III, 2015

Subject : STATISTICS (HONS.)

Paper : 301 (Gr. A + Gr. B)

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question.

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

Group A(a)

Answer **any two** out of four questions : 6x2=12

1. Compute mean and variance of Poisson Compound distribution. 6

2. State and prove Lindeberg-Levy Central Limit theorem. 2+4

3. The joint probability density function of X and Y is given by

$$f(x, y) = e^{-(x+y)},\ x > 0,\ y > 0.$$
$$= 0, \text{ otherwise}$$

Then find

(i) $P(X < Y \mid X < 2Y)$ and (ii) $P(1 < X + Y < 2)$ 6

4. Find Moment generating function of a bivariable Normal distribution $N_e(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.

Group A(b)

Answer **any two** out of four questions : 2x2=4

5. Let $\{x_n\}$be a sequence of random variables with mean (ε_R) cast variance 1. Show that $\frac{1}{n}\sum_{i=1}^{n} x_i \xrightarrow{P} \theta$, as $n \to \infty$.

6. Let $X_1, X_2, \ldots\ldots$ be i.i.d Poisson variables with parameter λ. Use Central limit theorem to find $P(120 \leq S_n \leq 160)$ where $S_n = X_1 + X_2 + \ldots\ldots + X_n)$, $n = 75$, and $\lambda = 2$.

7. Let (X, Y) follow bivariate Normal distribution with means 0 and variables 1 and correlation co-efficient p. Find regression line of Y and X.

8. State Chebychev's inequality and Weak law of large numbers.

Group B

Group B(a)

Answer **any one** out of two questions : 10x1=10

1. a) Write a short note on Gram-Schmidt Orghogonalization. 6

b) Find the determinant of $n \times n$ matrix : 4

$$\begin{pmatrix} 1+a & 1 & 1 & \cdots 1 \\ 1 & 1+a & 1 & \cdots 1 \\ 1 & 1 & 1+a & \cdots 1 \\ \cdots & \cdots & \cdots & \cdots \cdots \\ 1 & 1 & 1 & \cdots 1+a \end{pmatrix}$$

2. Find an orthogonal matrix P such that P^TAP is a diagonal matrix. 10

Group B(b)

Answer **any one** out of two questions : 6x1=6

3. a) Prove that the eigen values of a real symmetric matrix are all real. 2

   b) Show that $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$. 4

4. Find a basis and dimension of the subspace W of $\mathbb{R}^3$, where $W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$. 6

Group C

Answer **any four** out of eight questions : 2x4=8

5. A matrix A is a square matrix and $\lambda_1, \ldots \lambda_n$ be is eigen values. Find the Trace of A and det(A).

6. If A and B are Square matrix of order n, then show that

$$\det\begin{pmatrix} A & B \\ B & A \end{pmatrix} = \det(A + B)\det(A - B).$$

7. If λ be an eigen value of a real orthogonal matrix A, prove that $\frac{1}{\lambda}$ is also an eigen value of A.

8. Let A be a real symmetric matrix with distinct non-zero eigen values λ_1 and λ_2. Then show that corresponding eigen vectors are orthogonal.

9. Show that determinant of a Skew-symmetric matrix of odd order is zero.

10. State Cayley-Hamilton theorem. Verify it for I_3.

11. What do you mean basis and dimension of a vector space V.

12. Consider system of equations : $x + y + z = 6$; $2x + y + 3z = b + 1$; and $5x + 2y + az = b^2$, Determine the conditions for which above system of equations has unique solution.