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End Semester Examination of Semester–III, 2015

Subject : MATHEMATICS (HONS.) (UG)

Paper : VII (Theory)

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question.

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Mechanics (20 Marks)

Group A

Answer **any one** out of two questions : 10X1=10

1. a) Prove that angular momentum of a system of particles about the origin is equal to sum of the angular momentum of the centre of mass of the system about the origin and the angular momentum of he constituting particles about the centre of mass. 5

   b) What is Coriolis force? Discuss its effect on a freely falling body? 1+4

2. a) A particle of mass m moves in a central force field $V(r) = kmr^3$ $(k > 0)$. If its path is a circle of radius

a, (i) What is its period? (ii) What is its angular momentum? 2+3

b) State and prove the theorem of parallel axes regarding moment of inertia. 1+4

Group B

Answer **any one** out of two questions : 6X1=6

3\. a) A particle is moving along a straight line under an acceleration f, so that the distance traversed by it in time t is $x=\frac{1}{2}vt$, v being the instantaneous speed; find the relation between f and t. 2

b) Let $\vec{r}=x\hat{i}+y\hat{j}$ be the position vector of a particle with respect to a rotating orthogonal Cartesian frame, having unit vector $\hat{i}$ and $\hat{j}$ rotating in the anti clockwise direction with angular velocity ω. Taking $\frac{d\hat{i}}{dt}=\omega\hat{j}$, $\frac{d\hat{j}}{dt}=-\omega\hat{i}$ obtain the expressions for velocity and acceleration of the particle with respect to the above frame. 4

4\. a) For the function $\phi(x,y)=\frac{x}{(x^2+y^2)}$, find the magnitude at the directional derivative along a line making an angle 30^0 with the positive x-axis at (0, 2). 3

b) Show that the radial acceleration is two dimensional space is given by $|\vec{a}_r| = \frac{(x\ddot{x} + \ddot{y}y)}{\sqrt{x^2 + y^2}}$. 3

Group C

Answer **any two** out of four questions : 2x2=4

5. Derive the principle of angular momentum.

6. For a system of particles with masses m_i, position vectors $\vec{r}_i$ (i = 1, 2, 3, ..., n), define

   i) moment of momentum about the origin.

   ii) kinetic energy. 1+1

7. Write short notes on : Symmetrical top.

8. A particle moves from rest at a distance a from a centre of force where the repulsion a distance x is μx^{-2}. Show that its velocity at distance x is $\sqrt{\frac{2\mu(x-a)}{ax}}$.

Vector Calculus (20 Marks)

Group D

Answer **any one** out of two questions : 10x1=10

9. a) Let S be the surface $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + 2z = 2, z \geq 0\}$ and let $\hat{n}$ be the outward unit normal to S. If $F = y\hat{i} + xz\hat{j} + (x^2 + y^2)\hat{k}$ evaluate, $\iint_S \vec{F}.\hat{n}\,ds$.

   b) If $\vec{A}$ be a differentiable vector function and ϕ is a differentiable scalar function of position, prove that

   $$\vec{\nabla} \times (\phi\vec{A}) = (\vec{\nabla}\phi) \times \vec{A} + \phi(\vec{\nabla} \times \vec{A})$$

   Hence show that $\vec{\nabla} \times [(\vec{a}.\vec{r})\vec{a}] = \vec{0}$, $\vec{a} = a$ constant vector.

   3+2

10. a) Verify Green's theorem in the plane for

   $\int_C (2x - y^3)dx - xy\,dy$ where C is the boundary of the region enclosed by the circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. 5

   b) Using divergence theorem, evaluate $\iint_S \vec{A}.\hat{n}\,ds$, where $\vec{A} = 4xz\hat{i} - y^2\hat{j} + 4yz\hat{k}$, S is the surface of the solid bounded by the sphere $x^2 + y^2 + z^2 = 0$ and the paraboloid $x^2 + y^2 = z - 2$, and $\hat{n}$ is the outward unit normal vector to S. 5

Group E

Answer **any one** out of two questions : 6x1=6

11. a) For any space curve, prove that normal plane is orthogonal to the osculating plane. 3

b) If $\vec{A}=\vec{\nabla}\phi$ and $\nabla^2\phi=\rho$, a specific Scalar point function

show that $\iint\limits_S \frac{\partial\phi}{\partial x} ds = \iiint_v \rho dv$ 3

12. State Green's theorem in a plane and apply it to show that

$\frac{1}{2}\int\limits_C (x\,dy - y\,dx) =$ the area bounded by the curve C, where

C is a simple closed curve on a plane. 2+4

Group F

Answer **any two** out of four questions : 2x2=4

13. Let V be the volume enclosed by the surface S and $\hat{n}$ be the unit outward drawn normal to it, then prove that $\iiint_v \vec{\nabla}.\hat{n}\, dv = s$.

14. Show that $\vec{\nabla}\left[\vec{r}\,\vec{a}\,\vec{b}\right] = \vec{a}\times\vec{b}$, where $\vec{a}, \vec{b}$, where $\vec{a}$, $\vec{b}$ are constant vectors.

15. Prove the principle of energy by vector method.

16. If the vector $\bar{a}$ and $\bar{b}$ be irrotational, then show that the vector $\bar{a} \times \bar{b}$ is solenoidal.