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End Semester Examination of Semester–II, 2015

Subject : MATHEMATICS (HONS.)

Paper : III

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question.

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary

Group A (Mathematical Analysis–I)

Answer **any one** question from Q1 & Q2, one question from Q3 & Q4 and answer Q5 :

1. a) State LUB property of $\mathbb{R}$.

   Let $x \in \mathbb{R}$, show that there exists a unique $m \in \mathbb{Z}$ such that $m \leq x < m+1$. 1+4

   b) Let A be any set. Show that A is not equivalent to $\wp(A)$. 5

2. a) Test the Convergence of the Series : 5

   $a + b + a^2 + b^2 + a^3 + b^3 + \ldots\ldots\ldots$

   where $0 < a < b < 1$.

   b) Let $\{x_n\}$ be a sequence of real numbers such that $|x_{n+1} - x_n| \leq c\,|x_n - x_{n-1}|$ for some constant c with $0 < c < 1$. Show that $\{x_n\}$ is convergent. 5

3. a) Let $\{x_n\}$ be a convergent sequence with $\lim x_n = \ell$. Show that $\{|x_n|\}$ is also convergent with $\lim |x_n| = |\ell|$. Is the Converse true? 3

b) Does $\lim\limits_{x\to 0} \cos\frac{1}{x}$ exist? Justify your answer. 3

4. Let S be a non-empty bounded subset of $\mathbb{R}$ with Sups = M and infs = m. Prove that the Set $T = \{|x - y| : x \in S, y \in S\}$ is bounded above and SupT = M − m. 6

5. Answer **any two** : 2X2=4

a) Find $\text{lub}\left\{1-\frac{1}{n^2} : n \in \mathbb{N}\right\}$ 2

b) Determine J_n's such that $[0,1] = \bigcap\limits_{n \in \mathbb{N}} J_n$ where J_n's are open intervals containing [0, 1]. 2

c) Determine whether following statement is true or false : "If $\{x_n\}$ and $\{x_n y_n\}$ are bounded, then $\{y_n\}$ is bounded". 2

d) Let $0 \leq a < \epsilon$ for arbitrary positive ϵ. Prove that a = 0. 2

Group B (Differential Equations–I, Marks 20)

Answer **any one** question from Q6 & Q7, **one question** from Q8 & Q9 and answer Q10 :

6. a) Reduce the differential equation

$$\frac{dy}{dx} = 1 - x(y-x) - x^3(y-x)^3$$

to a linear form and hence solve it.

b) Show that the general solution of the equation

$$\frac{dy}{dx}+Py=Q$$

can be written in the form $y = k(u - v) + v$ where k is constant and u and v are its two particular solution.

5+5=10

7. a) Solve : $x\frac{dy}{dx}-y=(x-1)\left[\frac{d^2y}{dx^2}-x+1\right]$

[Changing by dependent variable]

b) Solve : $y=px+\sqrt{1+p^2}\,\phi\left(x^2+y^2\right),\ p=\frac{dy}{dx}.$

Where $\phi(x^2 + y^2)$ is a function of $x^2 + y^2$. 5+5

8. By the substitution, $x^2 = u$, $y^2 = v$ reduce the equation $x^2 + y^2 - (p + p^{-1})\, xy = c^2$ to Clairaut's form and find the general integral and singular solution. 2+2+2

9. a) Solve : $(mz - ny)p + (nx - lz)q = ly - mx$

where $p\equiv\frac{dz}{dx}$ and $q\equiv\frac{dz}{dy}$ respectively. 4

b) Find the orthogonal trajectories of the family of curves

$\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, λ being the parameter. 2

10. Answer **any two** : 2x2=4

a) Show that the necessary and sufficient condition for the ordinary differential equation Mdx+ Ndy = 0 to be exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

[assuming the functions M and N have continuous partial derivatives] 2

b) If $y=\sum_{n=0}^{\infty} a_n x^n$ be a series solution of the differential equation $\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+3y=0$

near the ordinary point x = 0 then show that

$n(n-1)a_n + (3n-3)a_{n-2} = 0$ for $n \geq 2$. 2

c) Consider the differential equation $(4x + 3y^2)dx + 2xydy = 0$, find an integrating factor of the form x^n, n being positive integer. 2

d) Evaluate : 2

$$\frac{1}{(D-3)(D-2)}\log x$$