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**End Semester Examination of Semester–I, 2015**

**Subject : MATHEMATICS (HONS.)**

**Paper : II (Theory)**

**Full Marks : 40**

**Time : 2 Hrs**

*The figures in the margin indicate the marks corresponding to the question*

*Candidates are requested to give their answers in their own word as far as practicable.*

*Illustrate the answers wherever necessary*

## Group A

1. Answer **any two** out of following four questions : 5X2=10

   a) i) A is a real square matrix and the matrix $(I + A)^{-1}(I - A)$ is skew symmetric, I being the identity matrix.

   Prove that A is orthogonal. $2\frac{1}{2}$

   ii) Obtain the fully reduced normal form of the matrix

   $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$. Find non-singular matrices P, a

   such that PAQ is in the fully reduced normal form.

   $2\frac{1}{2}$

b) i) If $\log\sin(\theta + i\phi) = \alpha + i\beta$ where $\theta, \phi, \alpha, \beta$ are real, prove that $2\cos 2\theta = e^{2\phi} + e^{-2\phi} - 4e^{2\alpha}$. 2

ii) Find the special roots of the equation $x^9 - 1 = 0$.

Deduce that $2\cos\frac{2\pi}{9}$, $2\cos\frac{4\pi}{9}$, $2\cos\frac{8\pi}{9}$, are the roots of the equation $x^3 - 3x + 1 = 0$. 3

c) If $\alpha, \beta, \gamma$ be the roots of $x^3 + 2Hx + G = 0$, prove that $(\alpha - \beta)^2 (\beta - \gamma)^2 (\gamma - \alpha)^2 = -27(G^2 + 4H^3)$ with the help of the determinant

$$\begin{vmatrix} 3 & \Sigma\alpha & \Sigma\alpha^2 \\ \Sigma\alpha & \Sigma\alpha^2 & \Sigma\alpha^3 \\ \Sigma\alpha^2 & \Sigma\alpha^3 & \Sigma\alpha^4 \end{vmatrix}$$ 5

d) i) If $\tan x = \frac{n\sin y}{1 - n\cos y}$ $(n<1)$ show that

$$x = n\sin y + \frac{1}{2}n^2 \sin 2y + \frac{n^3}{3}\sin 3y + \ldots\ldots\ldots\ldots$$ 3

ii) Show that the values of $i^i$ can be arranged so that they form a G.P. 2

2. Answer **any one** out of following two questions : 6X1=6

a) Describe the solution set in parametric vector form of the system $A\bar{X} = \bar{b}$, where $A = \begin{pmatrix} 1 & -3 & 2 \\ 7 & -21 & 14 \\ -3 & 9 & 6 \end{pmatrix}$,

$b = \begin{pmatrix} 4 \\ 28 \\ -12 \end{pmatrix}$. Also give the geometrical interpretation. 6

b) i) If $\alpha$ be a root of $x^3 - 3x + 1 = 0$, then show that other roots are $\alpha^2 - 2$ and $2 - \alpha - \alpha^2$. 3

ii) Show that the roots of the equation $x^7 = 1$ are the multiples of a, where $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$. Also prove that the roots of $x^2 + x + 2 = 0$ are $(a + a^2 + a^4)$ and $(a^3 + a^5 + a^6)$. 3

3. Answer **any two** out of following four questions : 2X2=4

a) Find mod z and amp z where $z = 1 + \cos 2\theta - i \sin 2\theta$, $\frac{\pi}{2} < \theta < \pi$.

b) If a, b, x are real and $|a + ib| = 1$, prove that $(a + ib)^{ix}$ is purely real.

c) Show that the equation $1+x+\frac{x^2}{2!}+\dots\dots+\frac{x^n}{n!}=0$ has no multiple root.

d) Show that each elementary matrix is non-singular.

## Group B

4. Answer **any two** out of following four questions : 5X2=10

a) Whatever origin and axis are chosen, show that the qualities $LX + MY + NZ$ and $X^2 + Y^2 + Z^2$ are invariants for any given system of forces.

b) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is stable.

c) Find the centre of gravity of a segment of a solid sphere of radius 'a' cut off by a plane situated at a distance 'c' (< a) from its centre.

d) A beam whose centre of gravity divides it into two portions 'a' and 'b' is placed inside a smooth sphere. Show that if $\theta$ be its inclination to the horizon in the position of equilibrium and $2\alpha$ be the angle subtended by the beam at the centre of the sphere then $\tan\theta = \left(\frac{b-a}{b+a}\right)\tan\alpha$



5. Answer **any one** out of following two questions : 6X1=6

a) Establish the conservation law of (i) Linear Momentum, (ii) Angular Momentum and (iii) Energy for the Mechanics of a particle. 3X2=6

b) Use Hamilton's equations, to find the equation of motion of the simple pendulum. 6

6. Answer **any one** out of following two questions : 2X1=2

a) Define cyclic co-ordinates.

b) When a body moves under the action of a system of conservation forces, prove that the sum of its kinetic and potential energy is constant throughout the motion.

7. Answer **any one** out of following two questions : 2X1=2

a) What are Wrench and Pitch?

b) If three coplanar forces acting on a rigid body are in equilibrium, then prove that they must be concurrent or parallel to one another.