Total Pages : 5

End Semester Examination of Semester–I, 2015
Subject : MATHEMATICS (HONS.)
Paper : I (Theory)
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary

Group A

Answer **any two** out of four questions : 10X2=20

1. a) If the three points on the parabola $\frac{\ell}{r}=1+\cos\theta$ with vectorial angles α, β, γ then show that the equation of circle circumscribing the triangle formed by the tangents at these points to the parabola is

$$r=\frac{\ell}{r}\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}\cos\left(\theta-\frac{\alpha+\beta+\gamma}{2}\right)$$

b) Show that the perpendiculars drawn from the origin to the normal planes of the cone $ax^2 + by^2 + cz^2 = 0$ generates the cone.

$$\frac{a(b-c)^2}{x^2}+\frac{b(c-a)^2}{y^2}+\frac{c(a-b)^2}{z^2}=0.$$ 5+5

2. a) Find the equation of the image of the line $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-4}{2}$ on the plane $2x-y+z+3=0$.

b) Find the locus of the poles of tangents to the director circle of an ellipse w.r.t. this ellipse. 5+5

3. a) Chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch the circle $x^2+y^2=c^2$, find the locus of their poles.

b) Find the sphere of the smallest radius that touches the straight lines

$$\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z-6}{1} \text{ and } \frac{x+3}{7}=\frac{y+3}{-6}=\frac{z+3}{1}.$$

5+5

4. a) Reduce the conic $x^2+4xy+4y^2+4x+y-15=0$ to its canonical form. 6

b) Find the equation of the cylinder whose guiding curve is $f(x, y)=0$, $z=0$ and generators are parallel to $\frac{x}{\ell}=\frac{y}{m}=\frac{z}{n}$. 4

Group B

Answer **any one** out of two questions : 6x1=6

5. If θ be the angle between two intersecting lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2; Show that the direction cosines of their angles bisectors are

$$\frac{\ell_1+\ell_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}} \text{ and } \frac{\ell_1-\ell_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}$$

6. Show that the value of k for which the plane $x+kz=1$ intersects the hyperboloid of two sheets $x^2+y^2-z^2+1=0$ in i) an ellipse, is $1<|k|<\sqrt{2}$.

ii) a hyperbola, is $|k|<1$. 3+3

Group C

Answer **any three** out of six questions : 2x3=6

7. Find the condition that the line $\frac{\ell}{r}=a\cos\theta+b\sin\theta$ may touch the conic $\frac{\ell}{r}=1+e\cos(\theta-\beta)$.

8. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.

9. Prove that the normals from the point (f, g, h) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone $\frac{f}{x-f}-\frac{g}{y-g}+\frac{a^2-b^2}{z-h}=0$.

10. The axes are rotated through an angle 60^0 without change of origin. The co-ordinates of a point are (4, √3) in the new system. Find the co-ordinates of it is the old system.

11. If PSP′, QSQ′ are two perpendicular focal chords of the conic $\frac{\ell}{r}=1+e\cos\theta$ then find the value of $\frac{1}{PP'}+\frac{1}{QQ'}$.

12. Show that no two generators of the same system of $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ intersect.

Group D

Answer **any one** out of two questions : 6X1=6

13. a) Solve the vector equation for $\bar{r}$:

$$t\bar{r}+\bar{r}\times\bar{a}=\bar{b}.$$

where $\bar{a}, \bar{b}$ are given vectors and t is a given non-zero scalar. 3

b) Show that the equation to the plane which contains the line $\bar{\gamma}=\bar{\alpha}+t\bar{\beta}$ and is perpendicular to the plane $\bar{\gamma}.\bar{\delta}=q$ is $\left[\bar{\gamma}-\bar{\alpha} \quad \bar{\beta} \quad \bar{\delta}\right]=0$. 3

14. Show that the three vectors $\bar{\alpha}\times\left(\bar{\beta}\times\bar{\gamma}\right)$, $\bar{\beta}\times\left(\bar{\gamma}\times\bar{\alpha}\right)$ and $\bar{\gamma}\times\left(\bar{\alpha}\times\bar{\beta}\right)$ are coplanar and find their mutual perpendicular vector. 6

Group E

Answer **any one** out of two questions : 2X1=2

15. Prove that $\left[\bar{b}\times\bar{c} \quad \bar{c}\times\bar{a} \quad \bar{a}\times\bar{b}\right]=\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$.

16. Find the scalar product of two vectors given by two diagonals of a unit cube. What is the angle between them? 1+1=2