Total Pages : 5

End Semester Examination of Semester–I, 2015

Subject : BCA

Paper : 1113 (Discrete Math)

Full Marks : 70

Time : 3 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

1. Answer **any five** questions : 2x5=10

a) Let R and S be the following relations on A = {1, 2, 3}. R = {(1, 1), (1, 2), (2, 3), (3, 1), (3, 3)}, S = {(1, 2), (1, 3), (2, 1), (3, 3)}, find R_0S.

b) Let R be a binary relation on the set of all positive integers such that R = {(a, b) : a – b is an odd positive integer} is R a symmetric relation?

c) If $a_0 = 2$, $a_1 = 3$ and $a_n = a_{n-1} + a_{n-2}$ then find the value of a_4, a_5.

d) Define simple graph with an example.

e) By means of truth table, show that

$\sim(p \Leftrightarrow q) = \sim p \Leftrightarrow q = p \Leftrightarrow \sim q$.

f) Let U be the set of all integers.
$A = \{x \in U : x^2 - 5x + 6 = 0\}$ and $B = \{x \in U : x^2 - 1 = 0\}$
Find (i) $A \cap B$ (ii) $A \cup B$ (iii) A' (iv) $A X B$.

g) Prove that product of two odd integers is an odd integers?

h) What is null graph give an example.

Group B

Answer **any five** questions : 5X4=20

2. If the function $f : R \rightarrow R$ be defined by $f(x) = x^2 + 1$ then find $f^{-1}(-8)$ and $f^{-1}(17)$.

3. Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with $a_0 = 1$ and $a_1 = -1$.

4. If a connected planner graph G has n vertices, e edges and r region then $n - e + r = 2$.

5. Prove the validity of the argument, If a number is odd, then its square is odd. K is a particular number that is odd, K^2 is odd.

6. Find the row canonical form of :

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{bmatrix}$$

7. If $f : A \to B$ and $g : B \to C$ are one to one functions, prove that $g_0 f : A \to C$ is one to one.

8. Give an example of (i) Bipartite graph, (ii) Petersen graph.

Group C

Answer **any four** questions : 4×10=40

9. a) The number of internal vertices in a binary tree is one less than the number of pendant vertices.

   b) In a class of 25 students, 12 have taken Mathematics 8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics and Biology and those who have taken Biology but not Mathematics.

10. a) If R is a relation from A to B and S is a relation from B to C. Show that $(R_0 S)^{-1} = S_0^{-1} R^{-1}$.

    b) Let $f : X \to Y$ be an everywhere defined invertible function and A and B be arbitrary now-empty subsets of Y, show that

    i) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

    ii) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$. 5+5

11. a) Write an equivalent formula for

    $P \wedge (q \Rightarrow r) \vee (r \Leftrightarrow p)$ which does not involve biconditional.

b) Let A={1, 2, 3} and let R and S be the relations on A such that

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

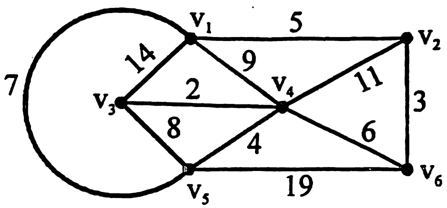
Find M_R^{-1}, $M_{R \cup S}$, $M_{R \cap S}$. 5+5

12. a) Draw the graph with the help of adjacency matrix :

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

b) Define a Hamiltonian graph and Eularian graph. Give an example of a graph which is Hamiltonian but not Eularian and vice-versa. 5+(2+3)

13. a) What is spanning tree? Using primes algorithm to find the Minimum weighted spanning tree for the following graph.



b) Prove that the number of pendant vertices in a binary tree is $\frac{x+1}{2}$ (where x is the number of vertices of tree).

6+4

14. a) A mapping $f : R \rightarrow R$ is defined by $f(x) = \frac{x}{1+x^2}, x \in R$ is one to one and onto. Find f^{-1} if exists.

b) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. (5+1)+4