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End Semester Examination of Semester–III, 2016

Subject : PHYSICS (PG)

Paper : PHSPG–301 (Theory)

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers whenever necessary

Use separate Answer scripts for Group A and Group B

Group A (Marks 20)

Answer Question No. 1 and any one out of Question No. 2 and Question No. 3.

1. Answer any five questions : 2x5=10

   i) Derive the equation of continuity for a particle obeying Dirac equation.

   ii) Prove that: $(\alpha.A)(\alpha.B)=A.B+i\sigma^{d}.(A\times B)$

   iii) Using the Born approximation calculate the differential scattering cross section for scattering by the central potential $V(r) = \frac{a}{r^2}$, where α is a constant. Given that $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$.

iv) Show that an attractive potential leads to positive phase shifts whereas a repulsive potential to negative.

v) Show that in the non-relativistic limit the Klein-Gordon equation reduces to the Schrödinger equation.

vi) Show that the Dirac Matrices can only be of even order and their values are ± 1.

vii) Under what conditions partial wave method and Born approximation method are applicable for theory of scattering in quantum mechanics?

viii) Define scattering length (a) for S-wave and show that the total cross-section is $4\pi|a|^2$.

2. a) Write down the expression of plane wave in terms of an infinite numbers of spherical waves and hence show that the scattering amplitude

$$f_K(\theta)=\frac{1}{K}\sum_{l=0}^{\infty}(2l\pi)\sin\delta_l\, e^{i\delta_l}P_l(\cos\theta).$$ 6

b) Show that Dirac theory predicts that the relativistic electron possesses spin $\frac{1}{2}$ only. 4

3. a) For an attractive square well potential, $V(r) = -V_0$ for $0 < r < r_0$ and $V(r) = 0$ for $r > r_0$. Find the energy dependence of the phase shift by born approximation. Hence show that at high energies,

$$\delta_0(k)\cong\frac{mr_0V_0}{h^2k}.$$ 4+1

b) For Dirac particle in central force field show that the total angular momentum $\left(\bar{L}+\frac{1}{2}\hbar\bar{\Sigma}\right)$ is a constant of motion.
[Symbols have their usual meanings] 5

Group – B (Marks 20)

Answer Question **No. 1** and **any one** out of Question **No. 2** and Question **No. 3**.

1. Answer any **five** questions: 2x5=10

i) Four distinguishable particles (spinless) are distributed among single particle energy levels having energies 0, 1, 2, 3... units. What would be the number of microstates considering the system four particles has total energy of 3 units?
Repeat the caculation if the system has identical spin zero and spin $\frac{1}{2}$ particles.

ii) Calculate the density of states in 3D, 2D and 1D as the functions of energy?

iii) Define mathematical and thermodynamic probability. What is their basic difference?

iv) 7×10^{10} particles are distributed among three energy levels in ratio 1 : 2 : 4. Show that the energy levels are equispaced.

v) A particle of mass m is confined in the region $0 < x < L$ where the rigid walls are placed at $x = 0$ and $x = L$. Draw the phase space trajectory of the particle and obtain the space volume with energy smaller than E.

vi) What is micro-canonical ensemble? Discuss the postulate of equal a priori probability.

vii) A classical gas of molecules each of mass m is in thermal equilibrium at the absolute temperature. Find the mean values of $v^2 v_x$ and $(av_x + bv_y)$ (where a and b are constants)

viii) How is the temperature of a system related to statistical entropy? Show with an example how the temperature can be negative.

2. a) A classical harmonic oscillator with $H = \frac{p^2}{2m} + \frac{1}{2}Kx^2$ is in thermal contact with a heat bath at temperature T. Calculate the partition function for the oscillator in the canonical ensemble and show explicitly that

$$\langle E \rangle = K_B T \text{ and } \left\langle \left(E - \bar{E}\right)^2 \right\rangle = K_B{}^2 T^2.$$

1+1+2

b) What is Gibb's paradox? Establish the Sackur Tetrode equation for an Boltzmann gas in microcanonical ensemble to show that it can resolve Gibb's paradox. 1+5

3. a) Prove that the mean value of an observable (F) in quantum statistics is represented by Trace $\left(\hat{P}\hat{F}\right)$. Where, $\hat{P}$ is the density matrix?

b) Show that the entropy

$$S=K_B \ln h(\mu, V, T)+K_B\beta\bar{E}-K_B\mu\beta\bar{N}$$

(Where the symbols have their usual significance) 2

c) Show that for ideal BE gas $PV - NK_BT \leq 0$ and for ideal FD gas $PV - NK_BT \geq 0$. Can you physically justify this difference for ideal BE and FD gas? 3+1