Total Pages : 5

**End Semester Examination of Semester–I, 2016**
**Subject : PHYSICS (PG)**
**Paper : PHSPG–102 (Theory)**
**Group : A & B**
**Full Marks : 40**
**Time : 2 Hrs**

*The figures in the margin indicate the marks corresponding to the question*

*Candidates are requested to give their answers in their own word as far as practicable.*

*Illustrate the answers whenever necessary*

***Use separate Answer scripts for Group A and Group B***

## Group A (Mark 20)

Answer **Q1** and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** questions: 2x5=10

a) Check whether the operator $-i\hbar x\frac{d}{dx}$ is hermitian.

b) Prove that

i) the projection operator $\hat{P}_i$ is an idempotent operator.

ii) $\hat{P}_i\hat{P}_j=\hat{P}_i\delta_{ij}$

c) The form of the wave function for the Hydrogen atom is given as

$$\psi_{nlm} = \frac{1}{162\sqrt{\pi}} \frac{r^2}{a_0^{\frac{7}{2}}} e^{\frac{r}{3a_0}} \sin^2\theta e^{-2i\phi}.$$

Find the quantum numbers *n, l, m* and parity.

d) Let $\phi_1$ & $\phi_2$ be two orthogonal eigen functions with energy eigen values 1 eV and 3 eV. A particle is in a state $\psi$ which is a super position of $\phi_1$ & $\phi_2$. The average value of energy is 2eV. Find the probablity of finding the particle in the states $\phi_1$ & $\phi_2$.

e) The quantum state of a two level system with energy (in eV) is given as $\psi(\vec{r},t) = 0.8\psi_1(\vec{r}) + 0.6e^{it/h}\psi_2(\vec{r})$. What are the energy values of the two states? Find the outcome of energy measurements (with probability) on the system.

f) A particle moving in one dimension is represented by a wave function $\psi(x) = Ae^{-|x|}$. Find the form of the potential in which the particle is moving.

g) Consider two states $|\phi_1\rangle$ and $|\phi_2\rangle$ with $|\phi_1|\phi_2\rangle = 0$. Show that these two states remain orthogonal under unitary transformation.

h) Write down simultaneous eigen functions of Hamiltonian and momentum for a free particle. Also construct simultaneous eigen functions of Hamiltonian and Parity for the free particle.

2. i) Let $\psi(x) = \begin{cases} \sqrt{\frac{2}{a}}\cos\frac{\pi x}{a} & for\ |x| \le \frac{a}{2}. \\ 0 & otherwise \end{cases}$

Calculate the uncertainty product. 5

ii) Let $\hat{A}$ & $\hat{B}$ be two Hermitian operators, satisfying the eigen value equations: $\hat{A}|u_n\rangle = a_n|u_n$ & $\hat{B}|u_n\rangle = b_\mu|v_\mu\rangle$. Find the transformation matrix which transforms the basis set $|u_n\rangle$ & $|v_\mu\rangle$. Show that the transformation matrix is unitary. 3

iii) The ground state wave function for hydrogen in coordinate space $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$ where $a_0 = \frac{\hbar^2}{me^2}$. Calculate the momentum wave function. 2

3. a) Write down Heisenberg equation of motion for a free particle of mass m moving in three dimensions. Hence evaluate
$[x_i(t = 0), x_j(t = 0)]$; $[x_i(t), x_i(t = 0)]$ (i, j = 1, 2, 3].
2+1+3

b) By what factors do the operators $\left(x^2 p_x^2 + p_x^2 x\right)$ and $\frac{1}{2}\left(xp_x + p_x x\right)^2$ differ? 4

## Group–B (Mark 20)

Answer **Q1**, and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** questions: 2x5=10

a) Find the ratio $\frac{c}{a}$ (c, a have usual meanings) and the packing fraction of hexagonal close packed structure.

b) Find the ratio of amplitudes of two types of atoms in a diatomic linear chain for acoustic and optical branches at

$K = 0$ and $K = \frac{\pi}{2a}$ ($2a$ = lattice constant)

c) Show that the anharmonic interactions in crystal can explain thermal expansion of crystal.

d) Derive the expression of the effective mass of an electron in an one-dimensional periodic potential. Just state the corresponding expression of reciprocal effective man tensor in three dimension.

e) Show that density of states becomes infinity at the zone boundary for a linear monoatomic lattice.

f) Determine the value of cut-off frequency assuming a linear lattice whose inter-atomic distance is given 3Å, the velocity of sound in this solid = 3 Km/sec.

g) In an assembly of $10^{23}$ SHO each has frequency of $10^{13}$ Hz. Calculate the mean energy of the system (ignoring zero point energy) at 20 K.

viii) Define a screw axis. Give a diagram showing a $z_1$, screw axis. 2

2. a) Write the Block function for electron states in a periodic potential. Show that the Block function is not eigen function of the momentum operator, but it is eigen function of lattice translation operator $T_{\vec{R}}$ ($\vec{R}$ = a lattice vector). 4

b) Find the total number of electron states in a band. Explain the differences between metals, semiconductors and insulators by band theory. Discuss also the overlapping of bands. 3

c) What is empty lattice approximation? Obtain the low-lying free electron energy bands of empty simple cubic lattice. 3

3. a) What is Debye-Waller effect? Find an expression of Debye-Waller factor. 1+3

b) Find the structure factor of a face centred cubic crystal and explain related conditions. 2

c) Explain why silver metal obeys Dulong-Petit law at room temperature but the diamond does not. Assume Einstein frequencies are $4 \times 10^{12}$ Hz for silver and $2.4 \times 10^{13}$ Hz for diamond. 2

d) What are meant by Normal process and Umklapp process? 2