Total Pages : 5

End Semester Examination of Semester–I, 2016

Subject : PHYSICS (PG)

Paper : PHSPG–101 (Theory)

Group : A & B

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question.

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer scripts for Group A and Group B

Group A (Mark : 20)

Answer **Q1**, and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** question: 2x5=10

i) State hypergeometric differential equation with parameters α, β, γ ($\gamma > 0$ and integral) leading to hyper geometric series as solution.

ii) For the complex function

$$f(z)=\frac{e^{\sqrt{z}}-e^{-\sqrt{z}}}{\sin\left(\sqrt{z}\right)}$$

Show that f(z) has a removable singularity at $z = 0$.

iii) Evaluate $\int_c \frac{z-5}{z^2-9z+18} dz$ around the circle $|z| = 1$.

iv) Prove that the set of vectors {(2, –3, 1), (4, 1,–5), (1, 1, 1)} is an orthogonal basis of the real inner product space R^3 with standard inner product.

v) Examine the following matrix

$$\begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

for diagonalizability.

vi) If erf(x) be the error function of x, show that

$$\int_0^\infty e^{-x^2-2bx}\, dx = \frac{\sqrt{\pi}}{2}.e^{b^2}\left[1-\text{erf}(b)\right].$$

vii) State (i) Schwarz inequality and (ii) Munkowski inequality for two functions f and g which are square integrable.

viii) If $(1 - 2xt + t^2) = \sum_{n=0}^{\infty} P_n(x)t^n$ for Legendre polynomial, then find the value of $P_3(-1)$.

2. a) Show that $y(x) = xe^x$ is a solution of the Volterra equation:

$$y(x) = \sin x + 2\int_0^x \cos(x-t)\, y(t) \sin t\, dt$$

4

b) Prove that the function f defined by

$$f(z) = \begin{cases} \dfrac{z^5}{|z|^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

satisfies the C–R equations at the origin, but is not differentiable at this point. 3

c) Solve Bessel differential equation and derive the value of $J_0(x)$. 3

3. a) Expand $\ln\frac{1+z}{1-z}$ in Taylor's Series about z = 0. 3

b) $\|x\|^2 \geq \sum_{i=1}^{n} |\langle x, v_i \rangle|^2$. 3

c) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ prove that $e^A = I + \frac{A}{3}(e^3 - 1)$. 4

Group–B (Mark 20)

Answer **Q1**, and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** questions: 2x5=10

i) A particle moving in a potential energy is given by $V(x) = bx^2 + \frac{a}{x^2}$, a, b > 0, find its spring constant and hence, frequency of oscillation.

ii) If a free particle moves between x_1 and x_2 in time t, find the action A for it.

iii) Prove that if the total force acting on the system of particles is zero then the total linear momentum will be conserved.

iv) For a contact transformation, prove that $\frac{\partial p_i}{\partial Q_j} = -\frac{\partial P_j}{\partial q_i}$

where (q_i, p_i) and (Q_j, P_j) are the old and new set of co-ordinates.

v) Prove that if F(q, p, t) and G(q, p, t) are two integrals of motion then [F, G] is also integral of motion.

vi) A system is governed by Hamiltonian

$$H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$$

a and b are constants p_x, p_y are momentum conjugate to x and y. Find the value of 'a' and 'b' so that the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved.

vii) A particle of mass m moves in a force field whose potential in spherical co-ordinates is

$$V = -\frac{K\cos\theta}{r^2}$$

Find the Hamilton Jacobi equation describing its motion.

viii) A bead moves on a circular wire. Specify the type of constraints.

2. a) A charged particle moves in a constant magnetic induction $\vec{B}$, so that $\vec{A}=\frac{1}{2}\vec{B}\times\vec{r}_x$. Find $[v_x, v_y]$. 3

b) State and prove least action principle. 2+5

3. Show that the Poisson Bracket of two dynamical variables remain invariant under canonical transformation.

Find the normal modes of vibration of freely vibrating linear triatomic molecules (neglect the interaction between the end atoms).

Consider a function F = F(q, p, t). Prove that the equation of motion of F in terms of Poisson brackets is

$$\frac{dF}{dt}=[F,H]+\frac{\partial F}{\partial t}$$

Where H is the Hamiltonian of the system. 4+4+2