Total Pages : 4

End Semester Examination of Semester–II, 2016

Subject : MATHEMATICS (PG)

Paper : MTMPG-204

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

Answer **any two** out of four questions : 10x2=20

1. a) Define finite complement topology on $\mathbb{R}$. Does this topology satisfy the Hausdorff axiom? The T_1 axiom? To what point or points does the sequence $\left\{\frac{1}{n}\right\}$ converge?

   b) Let X and Y be topological spaces and $f : X \to Y$. Show that f is continuous if and only if for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$. 6+4

2. a) Let X be any topological space and Y be an ordered set in the order topology. Let $f, g : X \to Y$ be continuous and $h : X \to Y$ be given by $h(x) = \min\{f(x), g(x)\}$. Show that h is continuous. 3

b) Let $f : A \rightarrow \prod_{\alpha \in J} X_\alpha$ be given by the equation

$$f(a) = (f_\alpha(a))_{\alpha \in J}$$

Where $f_\alpha : A \rightarrow X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Show that the function f is continuous if and only if each f_α is continuous.

Does the result hold if we use box topology? 4+3

3. a) Let $\Pi_1 : \mathbb{R}^2 \rightarrow \mathbb{R}$ be projection on the first coordinate. Let A = {(x, y) $\in \mathbb{R}^2$: x > 0 or y = 0 (or both)} be a subspace of $\mathbb{R}$. Let $q : A \rightarrow \mathbb{R}$ be obtained by restricting Π_1 on A. Show that q is a quotient map that is neither open nor closed. 5

b) Show that continuous image of a compact space is compact. 5

4. a) Prove that every compact subspace of a Hausdorff space is closed. 6

b) Define compact and locally compact spaces. Show by means of an example that locally compact space need not be compact. 2+2

Group B

Answer **any two** out of four questions : 6x2=12

5. a) Define quotient map and quotient topology. 2

   b) Construct torus from a rectangle topologically. 4

6. a) Define a normal space. 2

   b) Prove that every metrizable space is normal. 4

7. a) Define a connected topological space. 2

   b) Let X be a topological space. If {Ai} be a non-empty class of connected subspaces of X such that $\cap A_i$ is non empty then prove that $\cup A_i$ is also a connected subspace of X. 4

8. a) Define a completely regular space. 2

   b) Prove that product of completely regular spaces is completely regular. 4

Group C

Answer **any four** out of eight questions : 2x4=8

9. Define an open map with example.

10. Is it true that every compact subspace of a topological space is closed? Justify.

11. Show that in the finite complement topology on $\mathbb{R}$, every subspace is compact.

12. What do you mean by locally path connected space? Give an example.

13. Show that the set of all real numbers with the usual topology is first countable.

14. Give an example of a T_2 space which is not T_3.

15. Prove that the set of all rational numbers with the usual topology is not locally compact.

16. Show that the property of being a T_1 space is hereditary.