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End Semester Examination of Semester-II, 2016

Subject : MATHEMATICS (PG)

Paper : MTMPG-202

Full Marks : 40

Time : 2 Hrs

*The figures in the margin indicate the marks
corresponding to the question*

*Candidates are requested to give their answers
in their own word as far as practicable.*

Illustrate the answers wherever necessary

Group A

Answer any two out of four questions : 10x2=20

1. a) Describe a solution procedure to find the solution n linear equation by LU decomposition method.
- b) Consider a second order boundary value problem of the form

$$y''(x) + f(x) y'(x) + g(x) y(x) = r(x); \quad a < x < b$$

Where $f(x)$, $g(x)$, $r(x)$ are continuous functions of x or constants, with the boundary condition $y(a) = m_1$, $y(b) = m_2$. Apply finite difference method, show that the solution can be obtained from the equation $Ay = b$, where the matrices A and b are to be determined.

5+5

2. a) Prove that Crank-Nicolson method is unconditionally stable. 4

b) Consider $(n+1)$ data points (x_i, y_i) where $x_0 < x_1 < \dots < x_n$. Derive a piecewise cubic spline on $[x_0, x_n]$. 6

3. a) Determine the smallest eigen value and the associated eigen vector of the matrix $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ by power method. 5

b) Derive the Gauss-Legendre formula to evaluate
$$I = \int_a^b f(x) dx.$$
 5

4. a) Using the Chebyshev polynomials, obtain the least squares approximation of third degree for $f(x) = x^3$. $x \in [-1, 1]$. 5

b) What is ill-conditioned matrix? Find the condition number of the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1.01 \end{pmatrix}$.

Hence show that A is ill-conditioned. 2+2+1

Group B

Answer any two out of four questions : 6x2=12

5. Use the method of separation of symbols, prove that

$$f(2n) - 2 \binom{n}{1} f(2n-1) + 2^2 \binom{n}{2} f(2n-2) \dots + (-2)^n f(n) \\ = (-1)^n (c - 2an)$$

Where $f(x) = ax^2 + bx + c$.

6. Derive the relation between Chebyshev Polynomials of first kind and second kinds.

7. Derive the Simpsons' 1/3rd rule and modified Euler's formula from RK method of 4th order. What is the advantage of the RK method over Taylors series method. 3+2+1

8. Derive the diagonal five-point formula to solve the Laplace equation by finite difference method.

Group C

Answer any four out of eight questions : 2x4=8

9. If T_1, T_2 denote the trapezium rule approximation to

$$I = \int_a^b f(x) dx, \text{ which 1, 2 subintervals respectively, prove}$$

$$\text{that } (I - T_2) = \frac{1}{3} (T_2 - T_1).$$

10. Find Lagrange's polynomial for the function $\sin \pi x$, when $x = 0, \frac{1}{6}, \frac{1}{2}$. hence calculate $\sin \frac{\pi}{3}$.

11. Consider the function

$$f(x) = \begin{cases} -\frac{11}{2}x^3 + 26x^2 - \frac{75}{2}x + 18; & 1 \leq x \leq 2 \\ \frac{11}{2}x^3 - 40x^2 + \frac{189}{2}x - 70; & 2 \leq x \leq 3 \end{cases}$$

Show that $f(x)$ is a cubic spline.

12. Prove that the Chebyshev polynomial $T_n(\lambda)$ satisfies the differential equation

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + n^2y = 0$$

13. Is RK method a multistep method? Give reasons.
14. Derive the second order representation of the first derivative by forward difference as

$$u_x(ih, jk) = \frac{-3u_{i,j} + 4u_{i+1,j} - u_{i+2,j}}{h}$$

15. What is the concept of economization of power series?
16. Prove that, for a linear interpolation, for the case of equally spaced tabular data, the error does not exceed $\frac{1}{8}$ of the second difference.
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