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End Semester Examination of Semester–II, 2016
Subject : MATHEMATICS (PG)
Paper : MTMPG-201
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

Answer **any two** out of four questions : 10X2=20

1. a) Let H be a subgroup of a group G and N a normal subgroup of G. Show that NH is a subgroup of G and $N \cap H$ is a normal subgroup of H, Also show that $H/H \cap N \cong HN/N$.

   b) Show that the group Z_{60} has a composition series. 7+3

2. a) Show that any two composition series of G are isomorphic.

   b) Let G be a group of order p^2 where p is prime. Show that G is abelian. 7+3

3. a) Let G be a finite group and p a prime such that p^r divides $|G|$. Show that G contains a subgroup of order p^r.

   b) Define simple group. Show that no group of order 63 is simple. 6+(1+3)

4. a) State first hoarmorphism theorem for rings. Show that every epimorphism from the ring of integers $\mathbb{Z}$ into itself is an isomorphism. 2+3

   b) Let R be a commutative ring with unity. Show that every maximal ideal of R is a prime idea of R. Is the converse true? Justify your answer. 3+2

Group B

Answer **any two** out of four questions : 6x2=12

5. Let $P(x) = x^4 - 2x^3 + x + 1$. Show that p(x) is irreducible or Q[x].

6. If D is a UFD, then D[x] is a UFD.

7. Let E be an extension field of field F and $\alpha \in E$ with α algebraic over F. Show that there is a unique irreducible monic polynomial $p(x) \in F[x]$ of smallest degree such that $p(\alpha) = 0$. If f(x) is another monic polynomial in F[x] such that $f(\alpha) = 0$, then p(x) divides f(x).

8. Show that Every Euclidean domain is a principal ideal domain.

Group C

Answer **any four** out of eight questions : 2x4=8

9. Find all zeros of the polynomials $p(x) = 5x^3 + 4x^2 - x + 9$ in Z_{12}.

10. Find all of the units in Z[x].

11. Show that the polynomial $p(x) = x^3 + x^2 + 2$ is irreducible over $Z_3[x]$.

12. Show that every ideal in the ring of integers Z is a principal ideal.

13. Show that the group S_4 is solvable.

14. Let F be a field. Is F[x] a field? Justify your answer.

15. Is $\mathbb{Z}\left[\sqrt{-5}\right]$ a UFD? Justify your answer.

16. Suppose that $g^n = e$. Show that the order of g divides n.