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End Semester Examination of Semester–I, 2015

Subject : PHYSICS (PG)

Paper : PHS–102 (Theory)

Group : A & B

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers whenever necessary

Use separate Answer scripts for Group A and Group B

Group A (Mark 20)

Answer **Q1** and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** questions: 2x5=10

i) Find the energy values of a particle of mass m moving in a one dimensional potential

$$V(x) = \begin{cases} \infty & x \leq 0 \\ \frac{1}{2}m\omega^2x^2 & x > 0 \end{cases}$$

ii) Show that the expectation value of the operators, do not change under unitary transformation.

iii) Show that the commutator of two hermition operators is anti hermition.

iv) A particle is confined to box of length L with walls at x=0 and L. The normalized wave function of the particle at one instant is given as

$$\psi(x)=\sqrt{\frac{8}{5L}}\left[1+\cos\frac{\pi x}{L}\right]\sin\frac{\pi x}{L},$$ find the energy of the particle in this state.

v) The wave function of a state of the hydrogen atom is given by $\psi=3\psi_{200}+2\psi_{211}+\sqrt{2}\,\psi_{210}+3\psi_{21-1}$, where Ψ_{nlm} denotes the normalize eigen function of the state with quantum numbers n, l, and m in the usual notation. Find the expectation value of L_z in the state Ψ.

vi) Show that the parity operator commutes with the orbital angular momentum operator.

vii) Obtain the position operator in the momentum representation.

viii) Show that if P, Q and R are the operators in the schrodinger representation satisfying the relation [P, Q] = R, then the corresponding operators P_H, Q_H of the Heisenberg picture satisfy the relation $[P_H, Q_H] = R_H$.

Q2. a) A particle is in a state $\Psi(x)=\left(\frac{1}{\pi}\right)^{\frac{1}{4}} e^{-\frac{x^2}{2}}$. Evaluate the uncertainty product. 3

b) For a one dimensional SHO find $\langle n|\hat{x}^4|n\rangle$. 4

c) Consider a system whose state is given in terms of ortho normal set of basis vectors $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ as $|\Psi\rangle=\frac{1}{3}\left[\sqrt{3}|\phi_1\rangle+\frac{2}{3}|\phi_2\rangle+\sqrt{2}|\phi_2\rangle\right]$ consider an ensemble of 810 identical systems, each one of them in the state $|\Psi\rangle$. If measurements are done on all of them, How many systems will be found in each of the states $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$. 3

Q3. a) Find A, so that $\Psi(x, 0)$ is normalised. 2

b) If measurements of the energy carried out, what are the values that will be found, and what are the corresponding probabilities? Calculate the average energy. 1+1+2

c) Write the necessary form of wave function, dynamical operator and their time development in interaction Picture. 4

Group–B (Mark 20)

Answer **Q1,** and any **one** out of **Q2** and **Q3:**

Q1. Answer any **five** questions: 2x5=10

i) What are the symmetry operations in the point group 2 mm? Give the diagram (stereographic projection) illustrating the group.

ii) Find the reciprocal lattice of a simple cubic (sc) lattice.

Assuming $d_{hkl} = \frac{2\pi}{|\vec{G}hkl|}$, ($d_{hkl}$ = interplanar spacing of (hkl) planes, $\vec{G}hkl$ = reciprocal lattice vector), find the d_{111} of a sc lattice with lattice constant a.

iii) Take the potential energy of an anharmonic oscillator $V(u) = cu^2 - gu^3 - fu^4$ (u = displacement) with c, g, f all positive. Find the average displacement (u) (anharmonic potential terms small compared with K_BT, K_B = Boltzmann constant, T = temperature) applying Boltzmann statistics.

iv) Describe the extended, reduced and periodic zone schemes for describing band structure of a solid. Illustrate in one-dimensional case.

v) What is atomic factor? What does it signify?

vi) Define "effective mass of an electron". The energy versus wave-vector relationship for a conduction electron in a semi-conductor is $E=\frac{5\hbar^2k^2}{m_0}$. Determine the electron effective mass.

vii) What is Debye's T^3 law? How far is it satisfied by solids?

viii) Copper has fcc structure with atomic radius 0·1278 nm. calculate the interplanar spacing (321) planes.

Q2. a) Derive the wave vector–frequency dispersion relation of a monatomic linear chain with nearest neighbour interaction. Draw the curve. 3

b) Find the bands of electron energy in a Krönig-Penney peridic potential. Draw the E–K curve in the extended zone scheme. 7

Q3. a) Derive Lane equations for x-ray diffraction from a crystal. 4

b) What is Brillouin Zone? How can it be constructed using Bragg's diffraction condition? Draw grabs for the first, second and third Brillouin Zones in a one-dimensional lattice. 1+2

c) Suppose that the electron density for each of three electrons in natural Li can be represented by $C=\frac{e^{-\frac{2r}{a}}}{\pi a^3}$

Find out the atomic scattering factor for Li. 3