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End Semester Examination of Semester–I, 2015
Subject : PHYSICS (PG)
Paper : PHS–101 (Theory)
Group : A & B
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer scripts for Group A and Group B

Group A (Mark : 20)

Answer **Q1**, and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** question: 2x5=10

i) State and explain Jordan Curve theorem.

ii) Show that $f(z) = e^x(\cos y + i \sin y)$ is regular everywhere.

iii) Define with example isolated singularity of a complex variable f(z).

iv) If $X = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$, then find X^8 by Cayley-Hamilton's theorem.

v) Establish Cauchy-Riemann equations in polar form.

vi) Evaluate : $\int_C \frac{dz}{(z^2+4)^2}$, where C : $|z - i| = 2$.

vii) Find the residues of

$$f(z)=\frac{z^3}{(z-1)(z-2)(z-3)}$$ at infinity and $z = 1$.

viii) Find the value of $\Gamma\left(-\frac{3}{2}\right)$.

Q2. a) Show that $\frac{1}{\sin\left(\frac{\pi}{z}\right)}$ has non-isolated singularity at

$z=\pm\frac{1}{n}$. 3

b) Evaluate the integral : $\int_0^{1+i} z^2 dz$ 3

c) Use Cauchy's Integral formulae for evaluating the integral $\int_C \frac{\sin \pi z^2+\omega \pi z^2}{(z-1)(z-2)}$ where C is the circle $|z| = 3$.

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Q3. a) Find the eigen values and eigen functions of the matrix: 4

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) Prove that the function $f : \mathbb{C} \rightarrow \mathbb{C}$ defined by

$$f(z)=|z|, z \in \mathbb{C}$$

is nowhere differentiable in $\mathbb{C}$. 3

c) Prove that the eigen values of a Hermition matrix are all real and its eigen vectors corresponding to two distinct aigen values are orthogonal. 3

Group–B (Mark 20)

Answer **Q1**, and any **one** out of **Q2** and **Q3**:

Q1. Answer any **five** questions: 2x5=10

i) Show that if there exists a generating function F_1 (q, Q, t) such that

$$\frac{\partial F_1}{\partial t_1}(q,Q,t)=K-H.$$

Where K and H being the Hamiltonians in new set of Co-ordinates (Q, P) and old set of co-ordinates (q, p) then the transformation is canonical.

ii) Using Poisson's bracket show that

$$q(t,T)=q(t)+T[q,H]+\frac{T^2}{2!}[[q,H],H]+\frac{T^3}{3!}[[[q,H],H],H]+\ldots\ldots$$

iii) For a free particle motion, show that the Least Action principle coincides with the Fermat's Principle.

iv) If the Hamiltonian of a system does not involve time (t) explicitly, show that the generating functions S and W are then related to each other by the relation

$S(q, Q, t) = W(q, \theta) - \alpha_1 t$

where α_1 is a constant for separation of variable method.

v) Prove that the Hamiltonian H represents the total energy of the system E and is conserved, provided the system is conservative and T is a homogeneous quadratic function.

vi) The Lagrangian of a particle of mass m moving in a plane is given by

$$L = \frac{1}{2} m\left(v_x^2 + v_y^2\right) + a\left(x v_y - y v_x\right)$$

where v_x and v_y are velocity components and a is a constant. Determine the canonical momenta.

vii) Prove that a generalized co-ordinate cyclic in the Lagrangian is also cyclic in the Hamiltonian.

viii) For the Hamiltonian,

$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$

Show that $q_1 q_2$ = constant and $\frac{(p_2 - b q_2)}{q_1}$ = constant.

Q2. a) Using Poisson's bracket property, find the values of m and n so that the transformation

$$Q = q^m \cos(np),\ P = q^m \sin(np)$$

may represent a canonical transformation. Also obtain the generating function F_3. 4+2

b) Derive Hamilton's canonical equations from Hamilton's principle. 4

Q3. a) Write down Hamilton's canonical equation in terms of Poission's Bracket. 2

b) State and explain the Hamilton-Jacobi equation for Hamilton's principal function. 3

c) A particle is thrown horizontally from the top of a building of height h with an initial velocity u. Write down the Hamiltonian of the problem. Then determine the equation of motion of the particle using H–J method. 2+3