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End Semester Examination of Semester–III, 2015
Subject : MATHEMATICS (PG)
Paper : MTM–305 (IA) (Theory)
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question.

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary

Group A

Answer **any two** out of four questions : 10x2=20

1. a) Define Ricci tensor R_{ij} of first kind. 2

   b) Prove that it is symmetric. 3

   c) Find the scalar curvature of a sphere in E^3 with constant radius a. 5

2. a) Prove that $R^m_{ijk} + R^m_{jki} + R^m_{kij} = 0$. 4

   b) Define Einstein space. Show that for an Einstein space of dimension $n \geq 2$, $R_{ij} = \frac{r}{n} g_{ij}$ where r is the scalar curvature of the space. Prove also for an Einstein space, the scalar curvature is constant.

3. State and prove Sernet-Frenet formulae in space. 10

4. a) Find the differential equations of geodenic for the surface $x' = u \cos v$, $x^2 = u \sin v$, $x^3 = cv$, c being a constant. 5

   b) Find the Gaussian curvature of a sphere of radius r, with the parametric equation
   $x' = r \cos u \cos v$, $x^2 = r \cos u \sin v$, $x^3 = r \sin v$. 5

Group B

Answer **any two** out of four questions : 6X2=12

5. If $(a\lambda^r + b\mu^r + c\gamma^r)$ forms a parallel vector field along Γ,

   prove that $\frac{da}{ds} - kb = 0$, $\frac{db}{ds} + ka - \tau c = 0$; $\frac{dc}{ds} + \tau b = 0$,

   where the notations have their usual meanings. 6

6. Find the first fundamental form of a plane with respect to polar co-ordinates. 6

7. Show that the surface given by $x' = f_1(u')$, $x^2 = f_2(u')$, $x^3 = u^2$ is developble, where f_1, f_2 are differentiable functions. 6

8. Define Bertrand curves. Prove that for Bertrand curves the curvature and torsion are related by the relation $ak + b\tau = 1$. 6

Group C

Answer **any four** out of eight questions : 2x4=8

9. Show that in the V_4 with line element

$$ds^2 = -(dx')^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$$

the vector $\left(-1, 0, 0, \frac{1}{c}\right)$ is a null vector.

10. Calculate $\left\{ \begin{matrix} 1 \\ 2 \quad 2 \end{matrix} \right\}$ for the V_2 with line element

$$ds^2 = a^2(dx')^2 + a^2 \sin^2 x'(dx^2)^2$$

where a is a constant.

11. Prove that $\operatorname{div} A^i = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i}\left(\sqrt{g}\, A^i\right)$

12. Show that $x' = u' \cos u^2$, $x^2 = u' \sin u^2$, $x^3 = u' u^2$ is a regular surface.

13. Find the value of R^m_{mjk}.

14. When a space curve will be a plane curve? Justify your answer.

15. Prove that the normal curvature of a surface $K_{(n)} = \frac{\mathbb{B}}{\mathbb{A}}$, where $\mathbb{A}$ & $\mathbb{B}$ are first and second fundamental forms respectively.

16. When the two surfaces are called isometric? Give example.