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End Semester Examination of Semester–III, 2015

Subject : MATHEMATICS (PG)

Paper : MTM–304 (Theory)

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers whenever necessary

Group A

Answer **any two** out of four questions : 10x2=20

1. a) Prove that $\mathcal{L}\left\{\dfrac{e^{-\frac{a^2}{4t}}}{\sqrt{\pi t^3}}\right\}=\dfrac{2}{a}e^{-a\sqrt{s}}$ 5

   b) Using Fourier transformation, solve the integral equation

   $$\int_{-\infty}^{\infty}\frac{f(t)\,dt}{(x-t)^2+a^2}=\frac{1}{x^2+b^2}.$$ 5

2. a) Solve the integral equation

   $$f(x)=\phi(x)-\lambda\int_0^x e^{x-y}\phi(y)\,dy$$

   by the method of successive approximation 5

b) Solve the Laplace equation in the half-plane

$u_{xx} + u_{yy} = 0, \ -\infty < x < \infty, \ y \geq 0.$

with the boundary conditions :

$u(x, 0) = f(\lambda), \ -\infty < x < \infty$

$u(x, y) \to 0$ as $|x| \to \infty, \ y \to \infty.$ 5

3. a) If F(p) and G(p) denote the Laplace transform of f(t) and g(t) respectively, then show that F(p) G(p) is the Laplace transform of $\int_0^t f(s) g(t-s) ds.$ 4

b) Solve by Laplace transform technique, the equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, subject to the conditions $u(0, t) = 0 = u(5, t)$, $u(x, 0) = 10 \sin 4\pi x$ for $0 < x < 5, \ t > 0.$ 6

4. a) State Fredholm alternative theorem and use it to show that the integral equation

$$\phi(x) = f(x) + 2\int_0^1 (1 - 3xt)\phi(t) dt$$

will have a solution if $\int_0^1 (1-x)\phi(x) dx = 0$

b) Determine the eigenvalues and eigen function of the integral equation

$$\phi(x) = \lambda \int_0^{2\pi} \sin(x+t)\phi(t) dt$$ 6+4

Group B

Answer **any two** out of four questions : 6x2=12

5. Form the integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$$

with the given initial conditions x = y = 0, y' = –1, y″ = –5.

6. Solve the integral equation $u(x)=\cos x+\lambda\int_0^{\pi}\sin x.u(t)dt$

by separation of Kernals.

7. Show that $F\{H(ct-|x|)\}=\sqrt{\frac{\pi}{2}}\frac{\sin kct}{k}$.

8. Determine the values of λ for which the equation

$$\phi(x)=1+\lambda\int_0^{\pi}\cos(x+t)\phi(t)dt$$

has (i) a unique solution (ii) no solution and (iii) infinity many solutions. 6

[P.T.O.]

Group C

Answer **any four** out of eight questions : 2x4=8

9. Use Heaviside expansion theorem to evaluate

$$L^{-1}\left\{\frac{S}{S^2-3S+2}\right\}.$$

10. Prove the result $\sqrt{2\pi}\,\delta(x)*f(z)=f(z)$ for the convolution of Fourier transform.

11. Prove the shifting property of Fourier transforms.

12. Evaluate the integral $f(t)=\int_0^\infty \frac{\sin tx}{x\left(1+x^2\right)}dx$

by Laplace transformation method.

13. If F(u) is the Fourier transform of F(x) then show that $\frac{1}{a}F\left(\frac{p}{a}\right)$ is the Fourier transform of f(ax).

14. Find the inverse Fourier sine transform of $\frac{e^{-ap}}{p}$.

15. Define symmetric Kernel with example.

16. Convert the following differential equation into integral equation $y'' + y = 0$ where $y(0) = y'(0) = 0$.