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End Semester Examination of Semester–III, 2015
Subject : MATHEMATICS (PG)
Paper : MTM–302 (Theory)
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question.

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

Answer **any two** out of four questions : 10X2=20

1. a) Let (Ω, F, μ) be a measure space. Show that (i) μ is monotonic, (ii) μ is subadditivity, (ii) μ is continuity from below, (iv) μ is also continuity from above. 6

   b) Define Simple function. Let $f: \Omega \to [0, \infty]$ be measureable. Show that there exists a sequence of simple function $\{\phi_n\}$ on Ω with the property that $0 \le \phi_1(\omega) \le \phi_2(\omega) \le .... \le f(\omega)$ and $\phi_n(\omega) \to f(\omega)$, for every $\omega \in \Omega$.

2. a) Let $E \subset \Omega$. We define $\mu^*(E) = \inf \sum \mu(A_i)$, where the infimum is taken overall sequences of $\{A_i\}$ in A s.t.

$E \subset \cup A_i$. Let A* be the collection of all subsets $E \subset \Omega$ with the property that $\mu^*(F) = \mu^*(F \cap E) + \mu^*(F \cap E^c)$ for all sets $F \subset \Omega$. These two quantities satisfy show that

i) A* is a σ-algebra and μ* is a measure on A*.

ii) If $\mu^*(E) = 0$, then $E \in A^*$.

iii) $A \subset A^*$ and $\mu^*(E) = \mu(E)$, if $E \subset A$. 7

b) Let $\{f_n\}$ be a sequence for non-negative measureable function. Then show that

$$\int_{\Omega} \liminf f_n \, d\mu \leq \liminf \int_{\Omega} f_n \, d\mu.$$ 3

3. a) Let $f \in L'(\mu)$ and $\int_E f \, d\mu = 0$ for all measurable sets $E \subset \Omega$. Then show that $f = 0$ almost everywhere on Ω. 5

b) Let $E \subset X_x Y$ and define the projection $E_x = \{y \in Y : (x, y) \in E\}$ and $E^y = \{x \in X : (x, y) \in E\}$. If $E \in A X B$, then show that $E_x \in B$ and $E_y \in A$ for all $x \in X$ and $y \in Y$. 5

4. a) If X and Y are independent with $X \sim F_X$, $Y \sim G_Y$, then show that $X + Y \sim F * G$,

Where $F*G(z) = \int_R F(z-y) \, d\mu(y)$. 5

b) If $F_n \Rightarrow F$, then there exist random variables Y_n, Y with $Y_n \sim Y$ almost sequentially and show that $Y_n \sim F_n$ and $Y \sim F$. 5

Group B

Answer **any two** out of four questions : 6X2=12

5. What that the following are equivalent

   i) $X_n \Rightarrow X$.

   ii) For all open sets $G \subset R$, $\lim P(X_n \in G) \geq P(X \in G)$.

   iii) For all closed set $K \subset R$, $\lim P(X_n \in K) \leq P(x \in K)$.

6. A Roulette wheel has slots 1–38 (18 red and 18 black) and two slots 0 and 00 that are painted green. Players can be \$1 on each of the red and black slots. The player wins \$1 if the ball falls on his/her slot. Let $X_1, X_2, \ldots X_n$ be i.i.d with $X_i = \{\pm 1\}$ and $P(X_i = 1) = \frac{18}{38}$, $P(X_i = -1) = \frac{20}{38}$. Suppose we want to know $P(S_n \geq 0)$ after large number tries. 6

7. Define martingale process. Let $\{Z_i\}$, i = 1, 2, ... be a sequence of i.i.d. random variable with mean 0 and let $X_n = \sum_{i=1}^{n} Z_i$. Show that $\{X_n : n \geq 1\}$ is a martingle. 2+4

8. Establish Chapman-Kolmogorov equation by the transition probabilities of a Mankov chain.

Group C

Answer **any four** out of eight questions : 4X2=8

9. Find the differential equation of pure death process.

10. Define evolutionary process.

11. Suppose X_i are i.i.d. with $EX_i = \mu$, var $(x_i) < D$. Then show that $\frac{S_n}{n} \to \mu$ in L^2 and in probability .

12. Define complete measure space and Indicator function.

13. Let S be a semialgebra and let μ be defined on S. Suppose $\mu(\phi) = 0$ with the additional properties.

    If $E \in S$, $E = \bigcup_{i=1}^{n} E_i$, $E_i \in S$ disjoint, then show that $\mu(E) = \sum_{i=1}^{n} \mu(E_i)$

14. Define σ-Algebra.

15. Define measure and given an example.

16. Suppose F is a distribution function on R. Show that there is a unique measure μ on B(R) such that

    $\mu(a,b] = F(b) - F(a)$