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End Semester Examination of Semester–I, 2015
Subject : MATHEMATICS (PG)
Paper : MTM–105
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary

Graph Theory

Group A

Answer **any one** out of two questions : 10x1=10

1. i) Define components of a graph. Prove that for a simple graph with n vertices and k components can not have more than $(n-k)\ (n-k+1)/2$ edges. 1+4=5

   ii) For any Simple graph, prove that $\chi(G) \leq \Delta(G) + 1$, (Symbols have their usual meanings). 5

2. i) Define bipartite graph. In a bipartite graph with n vertices and m edges show that $m \leq \frac{n^2}{4}$. 1+2

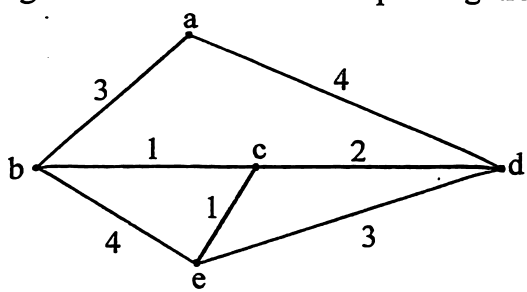
ii) Show that number of odd degree vertices in a graph is always even. 2

iii) Define Euler graph. Prove that a connected graph G is an Eular graph if and only if all vertices of G are of even degrees. 1+4

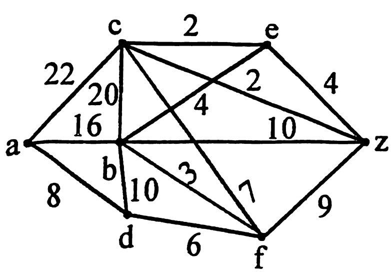
Group B

Answer **any one** out of two questions : : 6x1=6

3. Define spanning tree of the graph. Show how Kruskal's algorithm find a minimal spanning tree for the graph. 1+5



4. Use Dijktra's algorithm to find the shortest path between the vertices from a to z for the following graph. 6



Group C

Answer **any two** out of four questions : 2x2=4

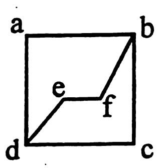
5. Define, with examples, isomorphic graphs.

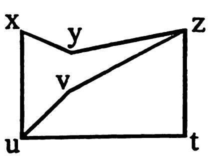
6. Draw the undirected graph corresponding to adjacency matrix :

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

7. Define perfect graph. Give the reason for which a bipartite graph $K_{m,n}$ $(m, n \geq 2)$ is perfect.

8. Check whether the following graphs are isomorphic or not.





(Discrete Mathematics)

Group D

Answer **any one** out of two questions : 10x1=10

9. a) Let S be a set containing n elements where n is a +ve integer. If γ is an integer such that $0 \leq \gamma \leq n$ then show that the number of sub sets S containing exactly γ elements is $\dfrac{n!}{\gamma!(n-\gamma)!}$

b) Let D_n denote the set of all positive divisor of n. If n = 30 then show that (D_{30}, '/') is a p_0 set where a/b means 'a divides b'. Also draw the Hasse diagram and determine it is a lattice or not. 5+5

10. i) Solve the recurrence relation :

$a_n = 6a_{n-1} - 9a_{n-2} + 2^n$. $(n \geq 2)$

with $a_0 = 1$, $a_1 = 0$. 5

ii) Define least and greatest element of a poset. Give an example of a poset with

a) a least element but no greatest element.

b) a greatest element but no least element. 1+2

iii) Consider the lattice L = (D_{12}, /), where D_{12} is the set of all divisors of 12. Find lower bound and upper bound of L. Is L a complemented Lattice? 2

Group E

Answer **any one** out of two questions : : 6x1=6

11. a) Let nth term a_n of the sequence $\{a_n\}$ satisfy the recurrence relation $a_n = 7a_{n-1} - 12a_{n-2} + 6, \ \forall \ n \geq 3$ with initial conditions $a_1 = 2, \ a_2 = 8$.

Prove that $a_n = 4^n - 3^n + 1. \quad \forall \ n \geq 1$.

b) Find the Transition diagram of the NFA (Non deterministic finite state automation) M with the Transition diagram Table–2 and find the language recognized by M.

	f	
s \ Σ	b	a
→ s_0	$\{s_1, s_2\}$	ϕ
s_1	s_1	s_2
s_2	ϕ	s_2

3+3

12. i) Describe the language L(G) for the grammar G = (N, T, P, S) where N = {A, B, S}, T = {a, b} and P consists of S→AB, AB→AAB, AB→ABB, A→a, B→b. What type of grammar is G?

ii) Build a FA accepting the language

$L = \{\omega \in \Sigma^*, \Sigma = \{a, b\} : \omega$ neither ends in ab or ba$\}$.

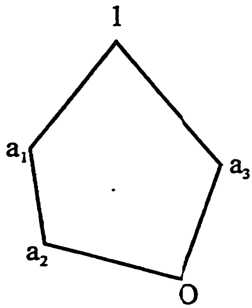
Also write the regular expression of L. 3+3

Group F

Answer **any two** out of four questions : 2x2=4

13. Find the minimum number of students in a class to be sure that four out of them are born in the same month.

14. Show that the lattice given below is not distributive.



15. Prove by Pumping Lemma that the language $L = 0^i1^i, i \geq 1$ is not regular.

16. Prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for any natural number n.