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End Semester Examination of Semester–I, 2015

Subject : MATHEMATICS (PG)

Paper : MTM–104

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

Answer **any two** out of four questions : 10x2=20

1. a) A function f(z) has a double pole z = 0 with residue 2; a simple pole at z = 1 with residue 2. It is analytic at all other points at finite distance from origin and is bounded at infinity. If f(2) = 5 and f(–1) = 2, find f(z). 5

   b) Show that every bilinear transformation with two distinct finite fixed points α, β ($\alpha \neq \beta$) can be put in the normal form.

   $$\frac{w-\alpha}{w-\beta}=\lambda\frac{z-\alpha}{z-\beta}$$

   Where $\lambda(\neq 0)$ being a complex parameter. 5

2. a) Discuss about branch cut and branch points of a multiple valued function $w = z^{\frac{1}{2}}$. 5

b) Use proper branch cuts and branch points and residue theorem to prove that

$$\int_0^{\infty} \frac{\ln x}{(x^2+1)^2} dx = -\frac{\pi}{4}$$ 5

3. a) Expand $f(z) = \frac{1}{z(z-1)}$ is a Laurent series valid for $1 < |z-2| < 2$. 5

b) Prove that an analytic function cannot be bounded in the neighbourhood of an isolated singularity. 5

4. a) Prove that $\sum_{n=-\infty}^{\infty} \frac{1}{n^2+a^2} = \frac{\pi}{a} \coth \pi a$, where $a > 0$. 5

b) Find an upper bound for the absolute value of $\oint_c \frac{e^z}{z+1} dz$, where c is the circle $|z| = 4$. 5

Group B

Answer **any two** out of four questions : 6x2=12

5. Prove by contour integration method

$$\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}; -1<a<1.$$ 6

6. Prove that a bounded entire function is a complex plane is constant.

   Hence derive the fundamental theorem of algebra. 3+3=6

7. Find the fixed points, nature of the bilinear transformation $\omega = \frac{(z+i)z-2}{(z+i)}$. Express it as a product of elementary transformations. 1+2+3=6

8. a) Show that under the transformation $w = z + \frac{1}{z}$ the upper semicircular disc with centre O and radius I is mapped onto the lower half plane. 3

   b) Show that the series $\sum_{n=0}^{\infty} \frac{z^n}{z^{n+1}}$ and $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(z-i)^{n+1}}$ are analytic continuation of each other. 3

Group C

Answer **any four** out of eight questions : 4x2=8

9. Let $f(z)=\frac{z^2+1}{\left(z^2+2z+2\right)^2}$. Evaluate $\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz$, where C is the circle $|z| = 4$. 2

10. Prove that $f(z) = \bar{z}$ is not a bilinear transformation. 2

11. Prove that an isolated singularity z_0 of $f(z)$ is a pole if and only if $\lim_{z\to z_0} f(z)=\infty$.. 2

12. Let $f(z)$ be an entire function such that for some k, $|f(z)| \le k|z|^3$ for $|z|\ge 1$ and $f(z) = f(iz)$ $\forall z \in \mathbb{C}$ prove that $f(z)$ is constant.

13. Using Rouche's theorem determine the number of zeros of the polynomial $P(z) = z^{10} - 6z^7 + 3z^3 + 1$ is $|z| < 1$.

14. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} 4^{(-1)^n . n} z^{2n}$.

15. Prove that $\int_0^{2\pi} \frac{d\theta}{1+\sin^2\theta} = \pi\sqrt{2}$ by Residue theorem.

16. Expand in Laurent series the function $f(z)=\frac{1-e^{2z}}{z^4}$ about the origin.