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**End Semester Examination of Semester–I, 2015**
**Subject : MATHEMATICS (PG)**
**Paper : MTM–103**
**Full Marks : 40**
**Time : 2 Hrs**

*The figures in the margin indicate the marks corresponding to the question*

*Candidates are requested to give their answers in their own word as far as practicable.*

*Illustrate the answers wherever necessary*

## Group A

Answer **any two** out of four questions : 10x2=20

1. a) Show that the motion of a particle in the potential field $V(r)=-\frac{k}{r}+\frac{h}{r^2}$ is the same as that of the motion under the Kepler potential alone when expressed in terms of a coordinate system rotating or processing around the center of force.

   b) A particle moves in the x–y plane under the constraint that its velocity vector is always directed towards a point on the x-axis whose abscissa is same given function of time f(t). Show that for differentiable f(t), the constraint is non-holonomic in general.

2. A Lagrangian for a particular physical system can be written as :

$$L=\frac{m}{2}\left(a\dot{x}^2+2b\dot{x}\dot{y}+c\dot{y}^2\right)-\frac{K}{2}\left(ax^2+2bxy+cy^2\right)$$

Where a, b and c are arbitrary constants but subject to the condition that $b^2 - ac \neq 0$. What are the equation of motion? Examine particularly the two cases $a = 0 = c$ and $b = 0$ and $c = -a$. What is the physical system described by the above Lagrangian? What is the significance of the condition on the value of $b^2 - ac$?

3. a) A rigid body is rotating about a fixed point at which A, A, C are principal moments of inertia under a couple $-\lambda\omega_1$, $-\lambda\omega_2$, $-\lambda\omega_3$ where $\omega_1$, $\omega_2$, $\omega_3$ are the angular velocity components under the principal axes. Prove that at time t,

$$\omega_1 = ae^{-\lambda t/A}\sin\left(\frac{\sigma c}{\lambda}e^{-\lambda t/C}+\epsilon\right)$$

$$\omega_2 = ae^{-\lambda t/A}\cos\left(\frac{\sigma c}{\lambda}e^{-\lambda t/C}+\epsilon\right)$$

$$\omega_3 = ne^{-\frac{\lambda t}{c}} \quad \text{where} \quad \sigma=\frac{n(C-A)}{A}$$

a, n are constants.

b) Also, show that instantaneous axes would approach ultimately to coincide with the least axis of the rigid body. 6+4

4. a) A particle of mass $m_1$ suspended from a fixed point O by a light in extensible string of length $l_1$ and a second particle of mass $m_2$ suspended from $m_1$ by another light inextensible string of length $l_2$. Calculate the frequencies of the double pendulum due to small oscillation.

b) Obtain the frequencies for the cases when (i) $m_1 >> m_2$, (ii) $m_1 << m_2$ and (iii) $m_1 = m_2$. 7+3

## Group B

Answer **any two** out of four questions : 6X2=12

5. Define constants of motion. Show that $F_1 = \frac{p_1 - aq_1}{q_2}$ and $F_2 = q_1q_2$ are constants of motion of a two degree of freedom described by the Hamiltonian $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$.

Are there any other independent algebraic constants of the motion?

6. Define cyclic co-ordinates. Derive the equations of motion with Routhian function for a dynamic problem having n degree of freedom of which k are cyclic co-ordinates. 1+5

7. Show that the transformation

$Q = \frac{1}{2}(q^2 + p^2)$, $P = \tan^{-1}\frac{q}{p}$ is canonical. Find the new

Hamiltonian function in (P, Q) where the old Hamiltonian is $H=\frac{1}{2}(p^2+q^2)$ and find the Hamilton's equation of motion in terms of new co-ordinates.

8. Is a body in the northern hemisphere falls freely to the ground from a height h, show that it strikes the ground at $\frac{2}{3}\omega h\left(\frac{2h}{g_e}\right)^{1/2}\cos\lambda$ to the east, where $\omega$ is the earth's angular velocity, $g_e$ is the acceleration due to combined effect of gravity and centrifugal force and $\lambda$ is the amplitude of the place.

## Group C

Answer **any four** out of eight questions : 4X2=8

9. State the principle of virtual work. What is the essence of the principle?

10. Show that time translation symmetry implies conservation of energy.

11. A dynamical system has the Lagrangian

$$L=\dot{q}_1^2+\frac{\dot{q}_2^2}{a+bq_1^2}+k_1q_1^2+k_2\dot{q}_1\dot{q}_2$$

where, a, b, $k_1$ and $k_2$ are constants. Find the Hamiltonian of the system.

12. Construct the Hamiltonian for a simple harmonic oscillator. Write down the equations of motion of the oscillator in phase space.

13. Define Poisson bracket. Show that if does not satisfy the commutative law of algebra. 2

14. Coriolis force does not contribute to the energy equation — Justify. 2

15. Write Hamilton-Jacobi's equations. What is the significance of the solution of this equation? 2

16. Write a brief note on moving frames of references. 2