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End Semester Examination of Semester–I, 2014

Subject : MATHEMATICS (PG)

Paper : 105 (Graph Theory & Discrete Analysis)

Full Marks : 40

Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers whenever necessary

Use separate Answer scripts for Group A and Group B

Group A (Graph Theory)

(Answer **any one** of the following) : 10x1

1. i) Define cut set and cut vertices of a connected graph. Show that the vertex connectivity of a graph is always less than or equal to the edge connectivity of the graph. 2+4

   ii) Define centre of a graph. Prove that every tree has either one or two centres. 1+3

2. a) Prove the following statements

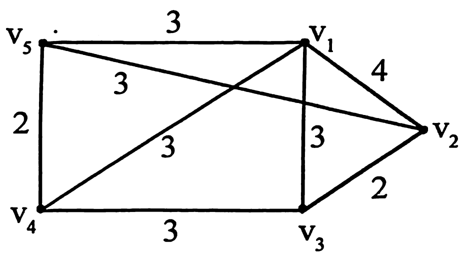
   i) A connected graph G is Eulerian if and only if it can be decomposed into edge-disjoint cycles.

ii) A graph G of order n (≥ 3) and size m if $m \geq \dfrac{n^2 - 3n + 6}{2}$ then G is Hamiltonian. 3+3

b) Define isomorphic graph. Draw three non-isomorphic graphs having same number of vertices, edges and vertices with equal degrees. 1+3

Answer **any one** of the following : 6x1

3. Find by Prim's algorithm a minimal spanning tree from the following graph : 6



4. Define binary tree? Prove that number of vertices in a binary tree is always odd? Show that the number of internal vertices in a full binary tree is one less than the number of pendent vertices. 1+2+3

Answer **any two** of the following : 2x2=4

5. Show that in any binary tree on n vertices, the number of pendent vertices is $\dfrac{(n+1)}{2}$. 2

6. Give an example of a graph which contains

   a) a Hamiltonian cycle but not an Eulerian Circuit.

   b) a Hamiltonian cycle and an Eulerian Circuit that are distinct. 1+1

7. If G is a self complementary graph of order n then n $\equiv$ 0 or 1 (mod 4). 2

8. Define unicursal graph with example. 2

Group B (Discrete Mathematics)

(Answer **any one** of the following) : 10x1

9. a) Solve the recurrence relation

   $a_{n+2} - 4\,a_{n+1} + 4\,a_n = 2^n$.

   b) For any positive integer n, let $I_n = \{x / 1 \leq x \leq n\}$ and $x \in I$ set of integers, let the relation "divides" be written as a/b iff a divides b. Show that $(I_{12}, /)$ is a poSet. Draw the Hasse diagram and determine it is a lattice or not. 5+5

10. a) Define distributive lattice. Give an example of a lattice that is not distributive. 4

    b) Show that the relation 'congruence modulo m' ($\equiv$) over the set of positive integers is an equivalence relation. 3

    c) How many solutions does $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2, x_3 are non-negative integers with $x_1 \leq 3$, $x_2 \leq 4$ and $1 \leq x_3 \leq 6$. 3

Answer **any one** of the following : 6x1

11. a) Show that if any eight positive integers are choosen two of them will have the same remainder when divided by 7.

b) If L be a lattice then for any a, b, c $\in$ L prove that $a \wedge (a \vee b) = a$. 3+3

12. a) For the NFA having state transition in table–1 find equivalent DFA

	0	1
q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_3\}$	ϕ
(q_3)	$\{q_3\}$	$\{q_3\}$

Table–1

b) Construct the regular grammar equivalent to the following NFA in fig 1. 3+3

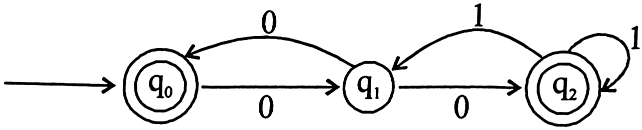
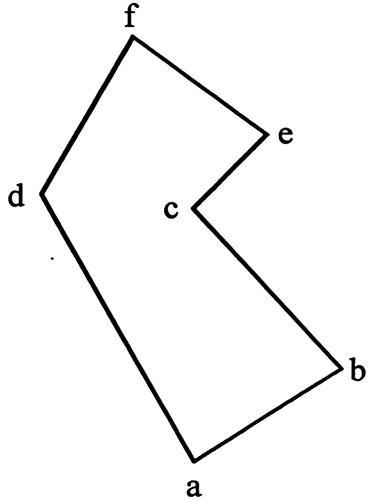


Fig 1

Answer **any two** of the following : 2x2=4

13. Derive the following Hasse diagram is a distributive lattice or not.



14. Let f : A → B and |A| > |B| then by Pi geonhole principle that f is not one to one.

15. A language is represented by a regular expression a * (a + ba). Which of the following string do not belongs to the regular set represented by the above expression.

    i) aaa ii) aba

    iii) aababa iv) aa

16. Prove that if L is accepted by a non deterministic finite automaton (NFSA) then L is also accepted by a DFA.