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End Semester Examination of Semester–I, 2014
Subject : MATHEMATICS (PG)
Paper : 104 (~~Real~~ Complex Analysis) (Theory)
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers whenever necessary

Group A

(Answer **any two** questions) : 10x2=20

1. i) State and prove Rouche's Theorem.

   ii) Use Rouche's theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$. 5+5

2. a) Show that, under suitable conditions, to be stated by you

   $$f'(a) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{(z-a)^2},$$

   where c is a closed contour surrounding the point z = a.

b) Prove by contour integration 5+5

$$\int_0^{\infty} \frac{x^{\alpha-1}}{1-x} dx = \pi \cos a\pi \qquad 0<\alpha<1$$

3. a) Show that the relation $w=\frac{iz+2}{4z+i}$ transforms the real axis in the z-plane into a circle in the w-plane. Find the centre and radius of the circle and the point in the z-plane which is mapped on the centre of the circle.

b) State and prove Riemann's theorem on removable singularities. 5+5

4. a) State and prove Mittag-Leffler's Theorem.

b) Show that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$

using the above theorem. 6+4

Group B

(Answer **any two** questions) : 6x2=12

5. Find the Mobius transformation which transforms the circle $|z| = 1$ onto $|\omega| = 1$ and makes the point $z = 1, -1$ corresponds to $\omega = 1, -1$ respectively. 6

6. State and prove Cauchy's residue theorem. 2+4

7. Find the radius of convergence of the power series.

i) $\sum \frac{n+1}{(n+2)(n+3)} z^n$

ii) $\sum \frac{n(\sqrt{2})+i}{1+2in} z^n$ 3+3

8. Evaluate **any one** of the following by the method of contour integration :

i) $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ 3

ii) $\int_{-\infty}^{\infty} \frac{x\cos x\, dx}{x^2+1}$ 3

Group C

(Answer **any four** questions) : 4x2=8

9. Find the radius of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{2^k z^{2k}}{k^2+k}$$

10. Find the Laurent Series expansion of $f(z)=\frac{1}{(z-1)(z-2)}$ centered at z = 1.

11. Give an example of removable singularity, why we call it removable?

12. State Schwarz Theorem.

13. Discuss the nature of singularities of the function $\frac{\sin z}{(z-\pi)^2}$.

14. Find bilinear transformation which maps 0, i, –i of z-plane to 1, –1, 0 of w-plane.