Total Pages : 3

End Semester Examination of Semester–I, 2014
Subject : MATHEMATICS (PG)
Paper : 103 (Theory)
Full Marks : 40
Time : 2 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers whenever necessary

Group A

(Answer **any two** questions) : 10x2=20

1. What do you mean by holonomic system? Deduce Lagrange's equations of motion for a unconnected holonomic system.

2. A double pendulum consists of two particles of mass m suspended by massless rods. Assume that all motion take place in a vertical plane. Find the Lagrangian of the system and derive the equations of motion of the system using Lagrange's equation of motion.

3. a) State and prove the necessary and sufficient conditions for a transformation to be canonical.

   b) Show that the transformation $Q=\log\left(\frac{1}{q}\sin p\right)$, P = q cotp is canonical.

4. A body moves under no forces about a point O, the principal moment of inertia at O, being 6A, 3A and A. Initially, angular velocity of the body has the components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = 3n$, about the principal axes. Show that at any time t,

   $\omega_2 = -\sqrt{5}\, n \tanh \sqrt{5}\, nt$ and ultimately body rotates about the mean axis.

Group B

(Answer **any two** questions) : 6x2=12

5. Prove that $J = \int_a^b F(y, y', x)\, dx$ will be minimum only when $\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0.$

6. What do you mean by Canonical transformation? Show that if the canonical transformation is given then one can determine the corresponding generating function.

7. Derive Euler's equations of motion for a rigid body.

8. Solve one dimensional harmonic oscillator problem by Hamilton-Jacobi's method.

Group C

(Answer **any four** questions) : 4x2=8

9. State and prove linear momentum conservation law for a system of particles.

10. If Hamiltonian of a system is not an explicit function of time then Hamiltonian is a constant of motion for the system.

11. What do you mean by non-inertial frame? Give an example of a non-inertial frame.

12. What do you mean by generalized coordinates and velocities?

13. What is Coriolis force? What is the cause of Coriolis force?

14. State Hamilton's principle.